Unsteady Flow of a Dusty Conducting Fluid between Parallel Porous Plates through Porous Medium with Temperature Dependent Viscosity

Rajesh Kumar

Abstract— **In this section we study, unsteady laminar flow of dusty conducting fluid between parallel porous plates through porous medium with temperature dependent viscosity. The fluid is considered as unsteady laminar flow and is acted upon by a constant pressure gradient and an external uniform magnetic field is applied perpendicular to the plates. It is assumed that the parallel plates are assumed to be subjected to a uniform suction from above and injection from below. It is also considered the viscosity is temperature dependent. The governing nonlinear partial differential equations are solved using finite difference approximation. The results for temperature field and velocity for both the dusty fluids and dust particles have been obtain numerically and displayed graphically.**

Index Terms— **Fluid mechanics, magneto hydrodynamics, heat transfer, transient state, two-phase flow, dust particles, finite differences. Porous medium**

I. INTRODUCTION

The flow of a dusty and electrically conducting fluid through a channel in the presence of a transverse magnetic field through porous medium has important application in various field such as magneto hydrodynamic generators, pumps, accelerators, cooling systems, centrifugal separation of matter from fluid, petroleum industry, purification of crude oil, electrostatic precipitation, polymer technology, and fluid droplets sprays. On the other hand, flow through porous medium have numerous engineering and geophysical applications for examples, in chemical engineering for filtration and purification process; in agriculture engineering to study the underground water resources; in petroleum technology to study the movement of natural gas, oil and water through the oil reservoirs, in view of these application, the object of the present paper is to study the effect of parallel plates through a porous medium with temperature dependent viscosity performance and efficiency of these devices are influenced by the present of suspended solid particles in the form of ash or soot as a result of the corrosion and wear activities and/or the combustion processes in MHD generators and plasma MHD accelerators.

The hydrodynamic flow of dusty fluids has been studied by P.G. saffman in (1962) on the stability of laminar flow of dusty gas. Soundalgekar, V.M. (1973) free convection effects on the oscillatory flow past on infinite vertical porous plates. Y.J. Kim (2000) unsteady MHD convective heat transfer past a semi-infinite vertical porous plate with variable suction. H.A. attia (2002) influence of temperature dependent viscosity on MHD coquette flow of dusty fluid with heat transfer. A. Ramanathan and G.suresh (2004) effect of megnatic field dependent voiscosity and anisotropy of porous medium ferro convection. Christopher J. seeton(2006) viscosity temperature correlation for liquid. M.F. EI-Amin,I. abbas, R.S.R gorla (2007) effect of thermal radiation on natural convection in aporous medium. H.A. attia (2008) unsteady hydromagnetic coquette flow of dusty fluid with temperature dependent viscosity and thermal conductivity. M.A Ezzat A.A.EI-Bary (2010) space approach to the hydro-magnetic flow of dusty fluid through a porous medium. D.S. chauhan, R. Agarwal (2011) MHD through a porous medium adjacent to a stretching sheet numerical and an approximation solution. Recently in year Unsteady flow of a dusty conducting fluid between parallel porous plates with temperature dependent viscosity.

In the present work, the effect on variable viscosity on the unsteady laminar flow of an electrically conducting, viscous, incompressible dusty fluid and heat transfer between parallel non-conducting porous plates is studied. The fluid is flowing between two electrically insulating infinite plates maintained at two constant but different temperatures. An external uniform magnetic field is applied perpendicular to the plates. The magnetic Reynolds number is assumed small so that the induced magnetic field is neglected. The fluid is acted upon by a constant pressure gradient and its viscosity is assumed to vary exponentially with temperature. The flow and temperature distributions of both the fluid and dust particles are governed by the coupled set of the momentum and energy equations. The joule and viscous dissipation terms in the energy equation are taken into consideration. The governing coupled nonlinear partial differential equations are solved numerically using the finite difference approximations. The effects of the external uniform magnetic field and the temperature dependent viscosity on the time development of both the velocity and temperature distributions are discussed.

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II. Description of the Problem

The dusty fluid is assumed to be flowing between two infinite horizontal plates located at the $y = \pm h$ planes, as show in Figure1. The dusty particles are assumed to be uniformly distributed throughout the porous plates. The two plates are assumed to be electrically non-conducting and kept at two constant temperatures: T_1 for the lower plate and T_2 for the upper plate with $T_2 > T_1$. A constant pressure gradient is applied in the *x*- direction and the parallel plates are assumed to be porous and subjected to a uniform suction from above and injection from below. Thus the y component of the velocity is constant and denoted by □$_0$. A uniform magnetic field B_0 is applied in the positive y- direction. By assuming a very small magnetic Reynolds number the induced magnetic field is neglected [15]. The fluid motion start from rest at *t*=0 and the no-slip condition at the plates implies that the fluid and dust particles velocities have neither a z nor an *x*-component at $y = \pm h$. The initial temperatures of the fluid and dust particles are assumed to be equal to T_1 and the fluid viscosity is assumed to vary exponentially with temperature. Since the plates are infinite in the x and z -direction, the physical variables are invariant in these directions. The flow of the fluid is governed by the Navier-Stokes equation[15].

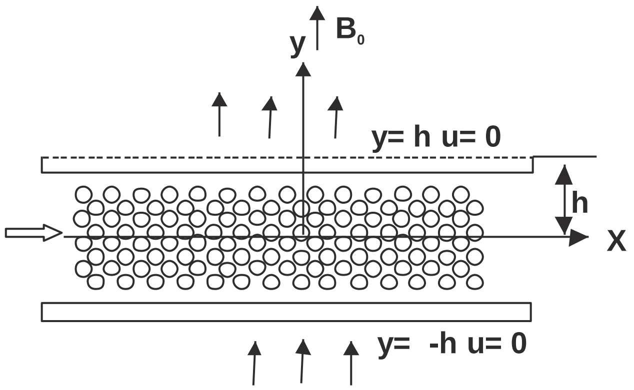

Figure 1. The geometry of the problem

$$\rho\frac{\partial u}{\partial t} + \rho v\frac{\partial u}{\partial y} = -\frac{dP}{dx} + \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) - \sigma B_\circ^2 u - \frac{\mu}{K_0'}u - KN\left(u - u_p\right) \quad ...(2.1)$$

Where ρ is the density of clean fluid, μ is the viscosity of clean fluid, *u*is the velocity of fluid, u_p is the velocity of dust particles, is the electric conductivity, *P* is the pressure acting on the fluid, *N* is the number of dust particles per unit volume, K_0' is porosity parameter and *K* is a constant. The first four terms in the right hand side are, respectively, the pressure gradient, viscous force ,porous medium and Lorentz force terms. The last term represents the force term due to the relative motion between fluid and dust particles. It is assumed that the Reynolds (number) the force term due to the relative velocity is small. In such a case the force between dust and fluid is proportional to the relative velocity [1]. The motion of the dust particles is governed by Newton's second law [1] via

$$m_p\frac{\partial u_p}{\partial t} = KN\left(u - u_p\right) \quad ...(2.2)$$

Where m_p is the average mass of dust particles. The initial and boundary conditions on the velocity fields are respectively given by

$$t = 0; \qquad u = u_p = 0 \quad ...(2.3)$$

For $t > 0$, the no-slip condition at the plates implies that

$$y = -h: \qquad u = u_p = 0 \quad ...(2.4)$$

$$y = +h: \qquad u = u_p = 0 \quad ...(2.5)$$

Heat transfer takes place from the upper hot plate towards the lower cold plate by conduction through porous medium the fluid. Also there is a heat generation due to both the joule and viscous dissipations. The dust particles gain heat energy from the fluid by conduction through their spherical surface. Two energy equations are required which describe the temperature distribution for both the fluid and dust particles and are respectively given by [16]

$$\rho c\frac{\partial T}{\partial t} + \rho c\upsilon_\circ\frac{\partial T}{\partial y} = k\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 + \sigma B_\circ^2 u^2 + \frac{\rho_p C_s}{\gamma T}\left(T_p - T\right) ....2.6$$

$$\frac{\partial T_p}{\partial t} = -\frac{1}{\gamma T}\left(T_p - T\right) \quad ...(2.7)$$

Where T is the temperature if the fluid, T_p is the temperature of the particles, c is the specific heat capacity of the of fluid at constant pressure, C_s is the specific heat capacity of the particles, k is the thermal conductivity of the fluid, γT is the temperature relaxation time $\left(=3\Pr\gamma_p C_s/2c\right), \gamma_p$ is the velocity relaxation time $\left(=2\rho_s D^2/3\mu\right)$, ρ_s is the material density of the dust particles $\left(=3\rho_p\mu/2D^2KN\right)$, D right-hand side of Eq. (2.6) represent the viscous dissipation, the joule dissipation, and the heat conduction between the fluid and dust particles, respectively. The initial and boundary conditions on the temperature fields are given as

$$t \le 0: \qquad T = T_p = 0, \qquad ..(2.8)$$

$$t > 0, \quad y = -h: \qquad T = T_p = T_1, \quad ..(2.9)$$

$$t > 0, \quad y = h: \qquad T = T_p = T_2, \quad ...(2.10)$$

The viscosity of the fluid is assumed to depend on temperature and is defined as $\mu = \mu_\circ f(T)$. By assuming the viscosity to very exponentially with temperature, the function f (T) Takes the form [13, 14] $f(T) = c^{-b(T-T1)}$, where the parameter b has the dimension of T^{-1} and such that at $T=T_1$, $f(T_1)=1$ and then $\mu = \mu_\circ$. This means that $\mu_\circ$ is the velocity coefficient at T= T_1. The parameter b may take positive values for liquids such as water, benzene or crude oil. In some gases like air, helium or methane a_1 may be negative, i.e. the coefficient viscosity increases with temperature [13, 14].

The problem is simplified by writing the equations in dimensionless form. The characteristic length is taken to be h, and the characteristic time is $\rho h^2/\mu_\circ$, while the characteristic velocity is $\mu_\circ/h\rho$. Thus we define the following non-dimensional quantities:

$$(\hat{x},\hat{y},\hat{z}) = (x,y,z)/h, \qquad \hat{t} = t\mu_\circ/\rho h^2,$$

$$\hat{P} = P\rho h^2/\mu_0^2, \quad \alpha = \frac{d\hat{p}}{d\hat{x}}$$

$$\frac{1}{K_0} = \frac{\mu h^2}{K_0^{'}\mu_0}, \qquad (\hat{u},\hat{\upsilon},\hat{w}) = (\hat{u},\hat{\upsilon},\hat{w})\rho h/\mu_\circ,$$

$$\left(\hat{u}_p,\hat{\upsilon}_p,\hat{w}_p\right) = \left(u_p,\upsilon_p,w_p\right)\rho h/\mu_\circ$$

$$\hat{T} = \frac{T-T_1}{T_2-T_1}, \qquad \hat{T}_p = \frac{T_p-T_1}{T_2-T_1},$$

$$f\left(\hat{T}\right) = e^{-b(T_2-T_1)\hat{T}} = e^{-a\hat{T}},$$

Where a is the viscosity parameter,

$$H_a^2 = \sigma B_0^2 h^2/\mu_\circ,$$

Where H_a is the Hartmann number, and

$$R = KNh^2/\mu_\circ$$

is the particle concentration parameter, $G = m_p\mu_\circ/\rho h^2 K$

is the particle mass parameter,

$$\varepsilon = \rho h\upsilon_\circ/\mu_\circ$$

is the suction parameter,

$$t \le 0, \qquad T = T_p = 1.$$

$$t > 0, \qquad y = 1;$$

$$\frac{\partial T}{\partial t} + \varepsilon\frac{\partial T}{\partial y} = \frac{1}{\Pr}\frac{\partial^2 T}{\partial y^2} + Ecf(T)\left(\frac{\partial u}{\partial y}\right)^2 EcH_a^2 u^2 + \frac{2R}{3\Pr}\left(T_p - T\right)$$

$\dfrac{\partial T_p}{\partial t} = -L_\circ\left(T_p - T\right)$ $\Pr = \mu_\circ c/k$ is the Prandtl number,

$$Ec = \mu_\circ^2 / \left(h^2 cp^2 \left(T^2 - T^1\right)\right)$$

is the Eckert number

$$L_0 = \rho h^2 / \mu_\circ \gamma T$$

is the temperature relaxation time parameters.

In terms of the above dimensionless variables and parameters, equation equations (2.1)-(2.6) take the following form (where we have dropped the hats for convenience):

$$\frac{\partial u}{\partial t} + \varepsilon \frac{\partial u}{\partial y} = a + f\left(T\right)\frac{\partial^2 u}{\partial y^2} + \frac{\partial f\left(T\right)}{\partial y}\frac{\partial u}{\partial y} - \left(H_a^2 + \frac{1}{K_0}\right)u - R\left(u - u_p\right)$$

…(2.11)

$$G\frac{\partial u_p}{\partial t} = \left(u - u_p\right)$$

…(2.12)

$$t \le 0; \qquad u = u_p = 0.$$

…(2.13)

$$t > 0, \qquad y = -1; \qquad u = u_p = 0,$$

…(2.14)

$$t > 0, \qquad y = 1; \qquad u = u_p = 0,$$

… (2.15)

$$\frac{\partial T}{\partial t} + \varepsilon \frac{\partial T}{\partial y} = \frac{1 \partial^2 T}{\Pr \partial y^2} + Ecf\left(T\right)\left(\frac{\partial u}{\partial y}\right)^2 EcH_a^2 u^2 + \frac{2R}{3\Pr}\left(T_p - T\right), \text{..........2.16}$$

…(2.16)

$$\frac{\partial T_p}{\partial t} = -L_\circ \left(T_p - T\right),$$

…(2.17)

$$t \le 0; T = T_p = 0$$

… (2.18)

$$t > 0, \qquad y = -1; \qquad T = T_p = 0,$$

… (2.19)

$$t > 0, \qquad y = 1; \qquad T = T_p = 1.$$

…(2.20)

Equations (2.11)-(2.12) represent a system of coupled and nonlinear partial differential equations which must be solved numerically under the initial and boundary conditions (2.13)-(2.15) and (2.18)-(2.20) using finite difference approximations [17]. The nonlinear terms are first linearized and then an iterative scheme is used at every time-step to solve the linearized system of difference equations.

III. Results and Discussions :

The exponential dependence of viscosity on temperature results in decomposing the viscous force term $= \frac{\partial}{\partial y}\left(\frac{\mu \partial u}{\partial y}\right)$ in Eq. (2.1) into two terms the variations of those resulting terms with the viscosity parameter a and their relative magnitudes have an important effect on the flow and temperature fields in the absence or is longer for *T* than for T_p, as T_p always follows *T*. It is noticed that the steady state values of T_p coincide with the corresponding steady state values of *T*, and the time required for T_p to reach the steady state, which depends on a, is longer than that for *T*. The reduction in temperature with increasing the viscosity exponent a that occurs at small time can be attributed to the fact that the only source term is the viscous dissipation (since $H_a = 0$). At small time the velocity gradient is small and an increase in a decreases the viscous dissipation as a result of decreasing viscosity and, in turn, decreases *T*. Numerical calculations have been carried out for dimensionless velocity of dusty fluid (*u*), velocity of dust particle (*v*) and temperature profiles *T* for different values of parameters which are displayed in Figures-(2.1) to (2.16).

Figures – (5.1), (5.2), (5.13) and (5.14) depict that with the increase in Hartmann (*Ha*) and concentration parameter (*R*) the velocity of fluid and dust particles decreases. This agrees with the natural phenomena because in the presence of magnetic field, Lorentz force sets in, which impedes the velocity of fluid and dust particles.

Figures – (5.3) and (5.4) depict that with increase in porosity parameter K_0 then increasing the value of the velocity of fluid and dust particles.

Figures – (5.5) and (5.6) depict that with increase in viscosity parameter *a* the velocity of fluid (*u*) and velocity of dust particles (*up*) increasing. The boundary layer and thermal boundary layer thicknesses reduce with increase in the viscosity parameter.

Figures – (5.7) and (5.8) depict that with increase in suction parameter of particle (ξ) increasing the value of velocity of fluid and dust particles because; increase in suction parameter of particle (ξ) reduces mass forces.

Form figures – (5.9), (5.10), (5.11), (5.12), (5.15) and (5.16) that the temperature profile of fluid and fluid particles is dreasing with increasing the value of suction parameter (ξ), Prandtl number (Pr) and concentration parameter (*R*). The boundary layer and thermal boundary layer thicknesses increase with increase the suction parameter (ξ), Prandtl number (Pr) and concentration parameter.

IV. Conclusion

In this paper the effect of a temperature dependent viscosity, suction and injection velocity and an external uniform magnetic field on the unsteady laminar flow and temperature distributions of an electrically conducting viscous incompressible dusty fluid between two parallel porous plate through porous medium has been studied. The viscosity was assumed to vary exponentially with temperature and the joule and viscous dissipations were taken into consideration. The most interesting result was the cross-over of the temperature curves due to the variation of the parameter *a* and the influence of the magnetic field in the suppression of such cross-over. On the other hand, changing the magnetic field results in the appearance of cross-over in the temperature curves for a given negative value of *a*. Also, changing the viscosity parameter a leads to asymmetric velocity profiles about the central plane of the channel (*y* = 0), which is similar to the effect of variable percolation perpendicular to the plates. The effect of the suction velocity on both the velocity and temperature of the fluid and particles is more pronounced for higher values of the parameter *a* and porosity parameter K_0.

V. Figures:

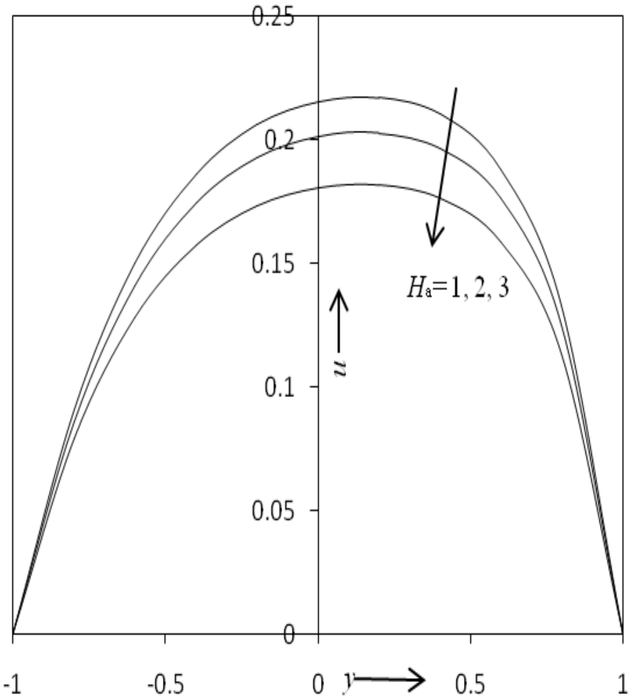

Fig. - 5.1: Velocity profile of fluid for different values of H_0

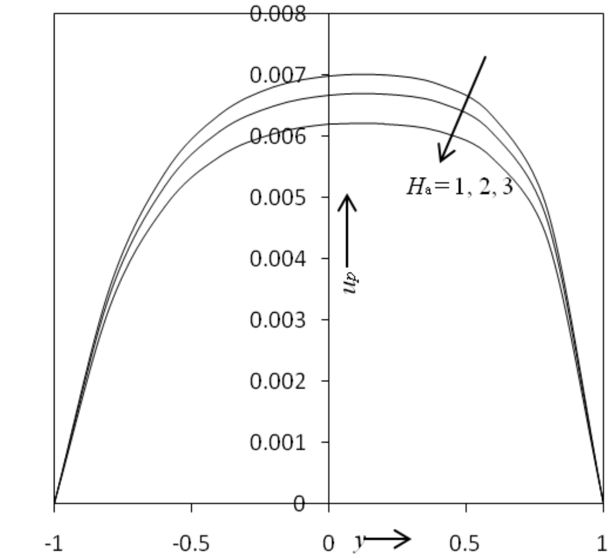

Fig. - 5.2: Velocity profile of dust particle for different values of H_0

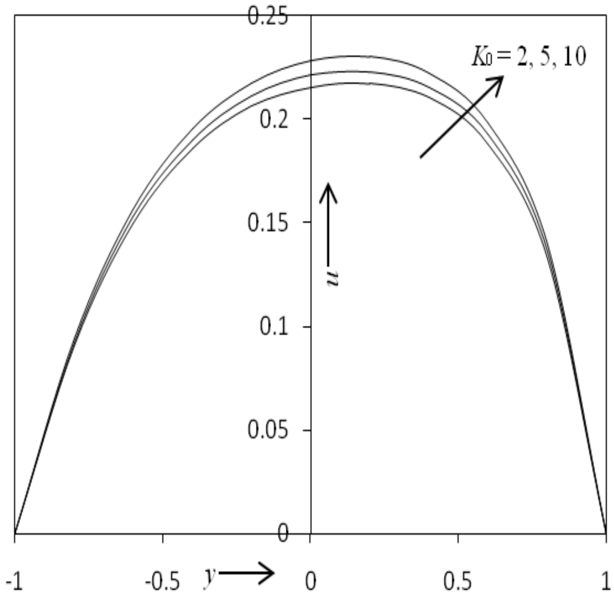

Fig. - 5.3: Velocity profile of fluid for different values of K_0

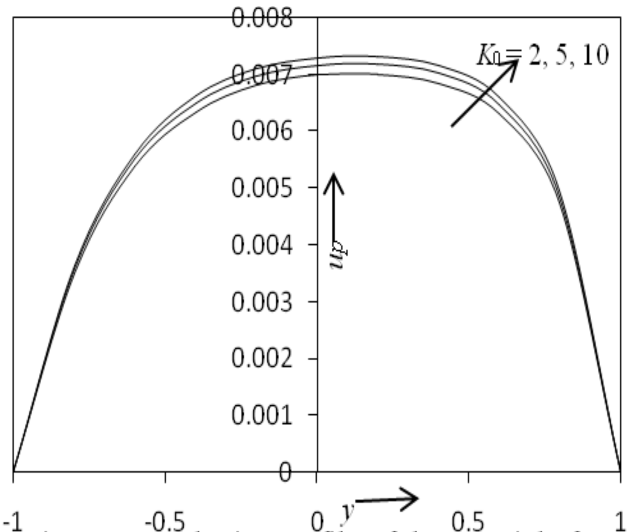

Fig. - 5.4: Velocity profile of dust particle for different values of K_0

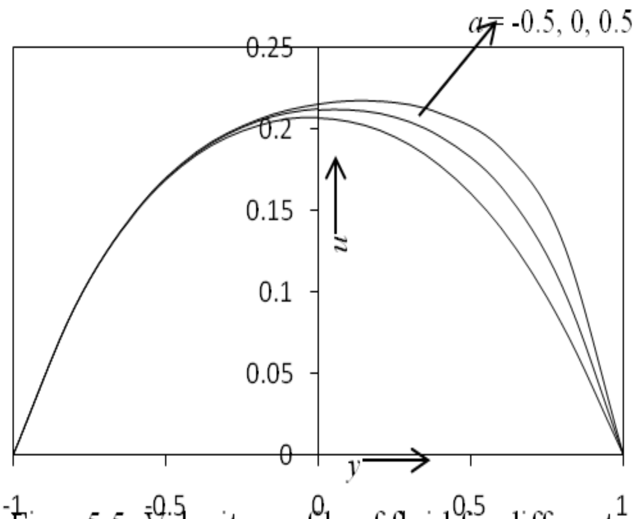

Fig. - 5.5: Velocity profile of fluid for different values of *a*

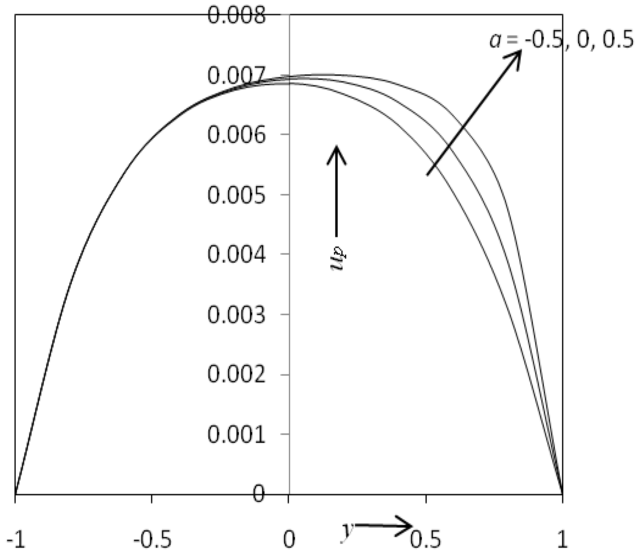

Fig. - 5.6: Velocity profile of dust particle for different values of *a*

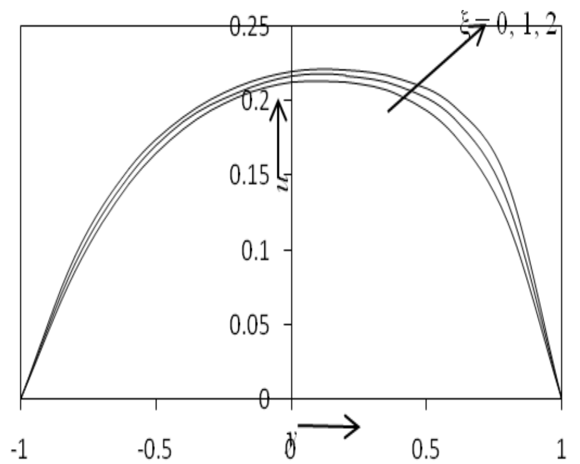

Fig. - 5.7: Velocity profile of fluid for different values of ξ.

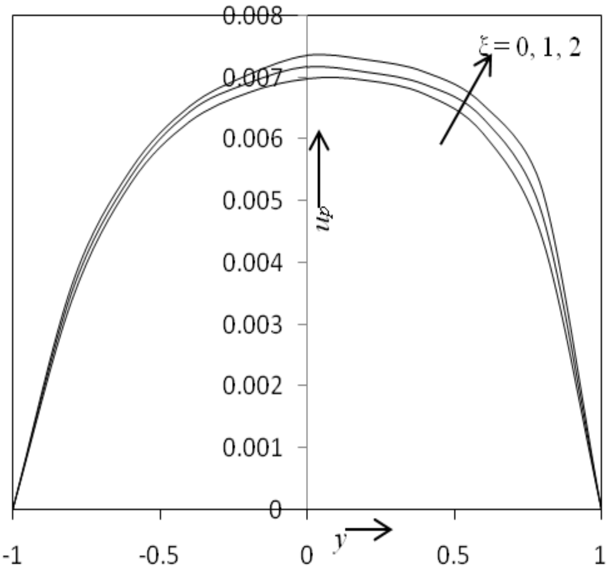

Fig. - 5.8: Velocity profile of dust particle for different values of ξ.

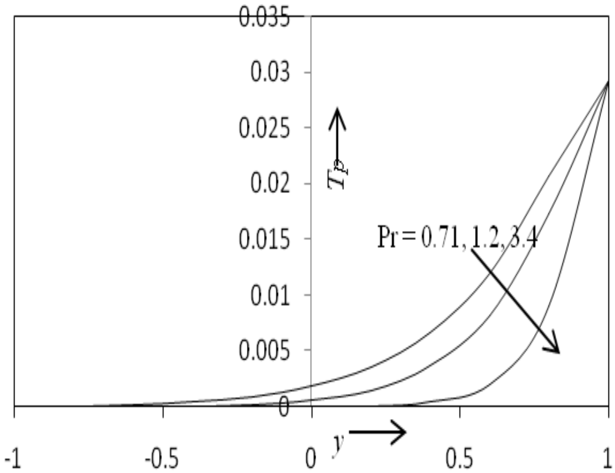

Fig. - 5.12: Temperature profile of dust particle for different values of Pr

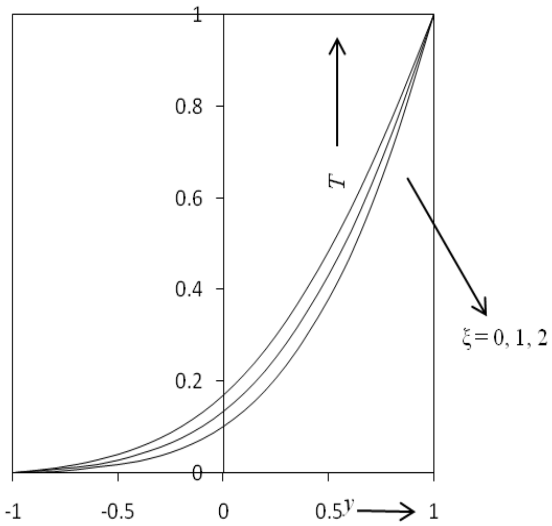

Fig. - 5.9: Temperature profile of fluid for different values of ξ.

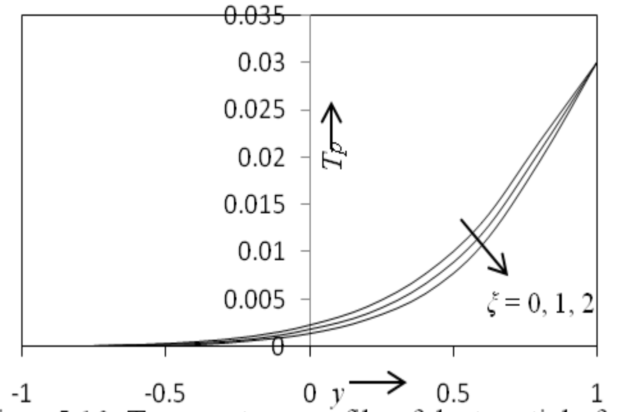

Fig. - 5.10: Temperature profile of dust particle for different values of ξ.

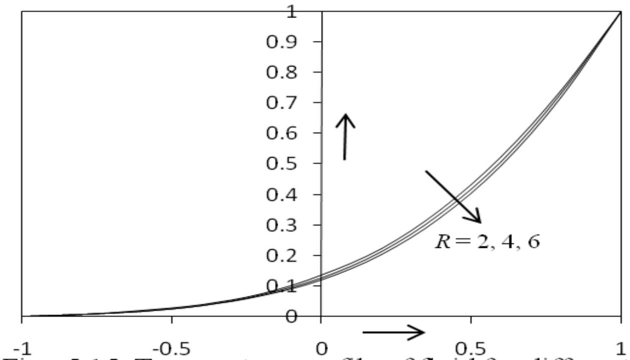

Fig. -5.15: Temperature profile of fluid for different values of R

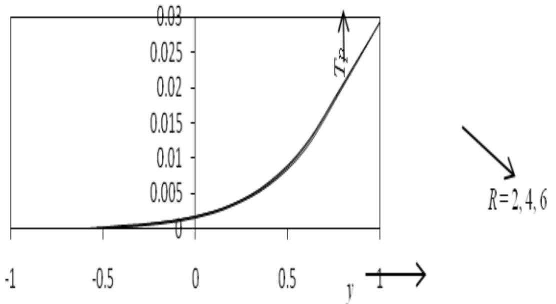

Fig. - 5.16: Temperature profile of dust particle for different values of R.

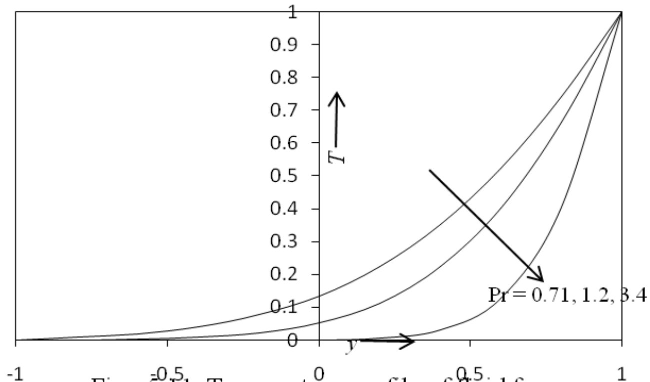

Fig. -5.11: Temperature profile of fluid for different values of Pr

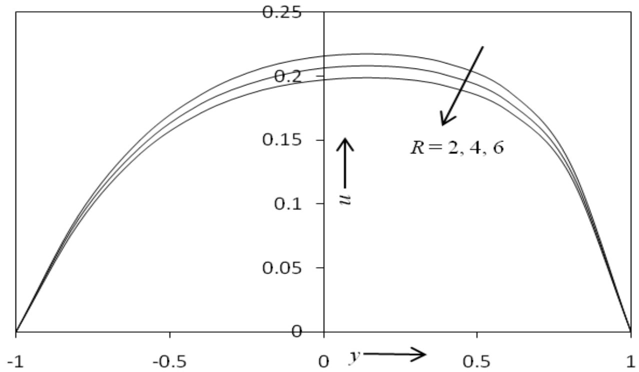

Fig. - 5.13: Velocity profile of fluid for different values of R

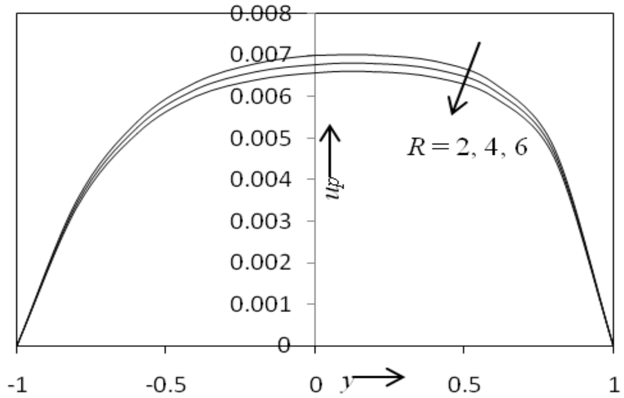

Fig. - 5.14: Velocity profile of dust particle for different values of R

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