

# Unsteady MHD Stratified flow of viscous Fluid Through Porous Medium Between Parallel plates

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**Abstract**— On hydromagnetic flow of a two phase fluid near a pulsating plate". In the present problem we have discussed MHD unsteady flow of stratified viscous fluid through porous medium between parallel plates.

**Index Terms**— MHD, Porous, viscous Fluid

## I. INTRODUCTION

It is worth mentioning that in the beginning of study of such flows, the effect of fluctuations of time dependent velocity is considered. The study of unsteady flow of a stratified fluid through a porous medium has been investigated by Ranganna and Channabasappa (1976) [5] and Gulab Ram & Mishra (1977) [6]. Gupta, P.C. & Prasad, M. (1987) [7] have also studied stratified viscous flow through a rectangular channel. The stratification effect on the Rayleigh layer over a naturally permeable wall have been investigated by Singh, A.K. and Sacheti, N.C. (1990) [8]. Das, D.K. and Nandy, K.C. (1993) [9] have investigated unsteady laminar stratified flow over a porous bed.

The Slip velocity for the flow of stratified fluid of variable viscosity past a porous bed under the action of pressure gradient has been discussed by Das, D.K. and Nandy, K.C. (1995) [10]. Singh, K.P. (1996) [11] has investigated unsteady flow of a stratified viscous fluid through a porous medium between two parallel plates with variable magnetic induction. Recently Ghosh S. and Ghosh A.K. (2005) [12] have discussed a problem entitled "On hydromagnetic flow of a two phase fluid near a pulsating plate". In the present problem we have discussed MHD unsteady flow of stratified viscous fluid through porous medium between parallel plates.

## II. BASIC EQUATIONS OF MOTION

Let  $x$  and  $z$  be the axes along the axis of the plate and the axis perpendicular to the plate respectively. We consider stratified viscous flow of an electrically conducting fluid through a porous medium of absolute permeability, bounded by two parallel plates in the presence of transverse magnetic field.

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$$u' = \frac{\lambda}{\mu_0 M} u; \quad t' = \frac{\rho_0}{\mu_0} t \quad \dots (3.6)$$

We take the boundary conditions as follows:

$$u = (1 + e^{i\alpha t}) \text{ at } z = 0$$

$$u = 0 \text{ at } z = d \quad \dots (3.7)$$

where  $\alpha$  = frequency of fluctuations.

With the help of equation (3.6), the equation (3.5) after dropping dashes is reduced to the form:

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial z^2} + \alpha u \quad \dots (3.8)$$

In accordance with the new technique of Lighthill the above equation can be solved by considering the velocity as the sum of two parts one time dependent and another as time independent, therefore we have  $u$  as given below:

$$u = a_1(z) + e^{i\alpha t} a_2(z)$$

From equations (3.8) and (3.9) we obtain,

$$e^{i\alpha t} \left[ \nu \frac{d^2 a_2(z)}{dz^2} - \alpha a_2(z) \right] + M^2 \pm e^{i\alpha t} \frac{d^2 a_1(z)}{dz^2} = 0 \quad \dots (3.11a)$$

Separating equation (3.10) into time dependent and time independent parts, we obtain,

$$\nu \frac{d^2 a_1(z)}{dz^2} - M^2 a_1(z) = 0 \quad \dots (3.11b)$$

In the light of equation (3.9), the boundary conditions given by (3.7) come out to be in the following form:

$$a_1(z) = 1 \text{ at } z = 0, \quad a_2(z) = u_0 \text{ at } z = d,$$

...

$$a_2(z) = 1 \text{ at } z = 0, \quad a_2(z) = 0 \text{ at } z = d, \quad \dots (3.13)$$

Finding out the solution of equation (3.11a), we have

$$a_1(z) = \frac{1}{2} \left[ \frac{e^{-Mz}}{M} + \frac{e^{Mz}}{M} \right] = \frac{\cosh(Mz)}{M} \quad \dots (3.14)$$

Now constants  $A_1$  and  $A_2$  under conditions (3.12) are computed as follows:  $A_1 = \frac{1}{2} (4 - 1) / 2d^{3/2} \sinh \left[ \frac{3}{2} \right]$

$$4M^2) d/2] \dots (3.15)$$

$$A2 = \{(40 - 1)/2e^{13(112)} \sinh [(13^2 + 4M^2) d/2] \dots (3.16)$$

we have plotted the graphs to represent the effect of Hartmann number  $M$  and stratification factor  $\rho$  on the velocity distribution. The different parameters have been taken as given under :

$$co = 0.5, d = 2.0,$$

$$u = 12 \text{ and } E = 0.1.$$

The graphs ( $G_1$ ), ( $G_2$ ), ( $G_3$ ) and ( $G_4$ ) have been plotted for different values of Hartmann number  $M$ , against the different values of stratification factor  $13$  :

The graphs ( $G_2$ ) and ( $G_3$ ) represent that the velocity decreases with the increase in the value of  $\rho$  for given  $M$ . From the graphs ( $G_1$ ), ( $G_2$ ) and ( $G_3$ ) it is clear that the velocity decreases with the increase in  $M$  for the given value of stratification factor  $\rho$ .

From the graphs ( $G_3$ ) and ( $G_4$ ), it is obvious that the motion of the fluid decelerates with increase in the values of  $\rho$  and  $M$ .

As such from the aforesaid results it is established that the induced magnetic field and stratification parameters are held responsible for retardation in the motion of the fluid.

At  $t = 0$ ,

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The governing equations of motion are :

$$\frac{\partial p}{\partial z} = -g \dots (2.1)$$

$$\frac{\partial}{\partial x} (\rho u) = 0 \dots (2.2)$$

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho} \left( \frac{\partial p}{\partial x} \right) + \frac{1}{\rho} \cdot \frac{\partial}{\partial z} \left( \mu \frac{\partial u}{\partial z} \right) - \frac{\mu u}{\rho k} - \frac{\sigma}{\rho} (B^2 u) \dots (2.3)$$

- where,  $p$  = Pressure of the fluid,
- $u$  = Velocity of the fluid motion,
- $t$  = function of time,
- $\rho$  = density of the fluid
- $\mu$  = Viscosity of the fluid,
- $K$  = porosity of porous medium,
- $B$  = magnetic induction.

MATHEMATICAL ANALYSIS

We assume that

$$B = B_0 e^{-\beta z/2} \dots (3.1)$$

$$\rho = \rho_0 e^{-\beta z} \dots (3.2)$$

$$\mu = \mu_0 e^{-\beta z} \dots (3.3)$$

- where,  $B_0$  = Magnetic induction on the fluid at  $z = 0$ ,
- $\rho_0$  = Density of the fluid at  $z = 0$ ,
- $\mu_0$  = Viscosity of the fluid at  $z = 0$ ,
- $\beta (\beta > 0)$  = The stratification factor at  $z = 0$ .

We also assume that the pressure gradient is in the form :

$$\frac{\partial p}{\partial x} = \lambda e^{-\beta z} \dots (3.4)$$

By virtue of equations (3.1), (3.2), (3.3) and (3.4), the equation (2.3) becomes :

$$\frac{\partial u}{\partial t} = \frac{\mu_0}{\rho} \cdot \frac{\partial^2 u}{\partial z^2} - B \frac{\mu_0}{\rho} \cdot \frac{\partial u}{\partial z} - \frac{\mu_0}{\rho} \left( \frac{1}{K} + \sigma \frac{B_0^2}{\mu_0} \right) u + \frac{\lambda}{\rho} \dots (3.5)$$

Now we introduce the following non-dimensional quantities as given below :

$$z' = z; \quad M^2 = \left( \frac{1}{K'} + \frac{B^2}{\mu_0} \right); \dots (3.6)$$

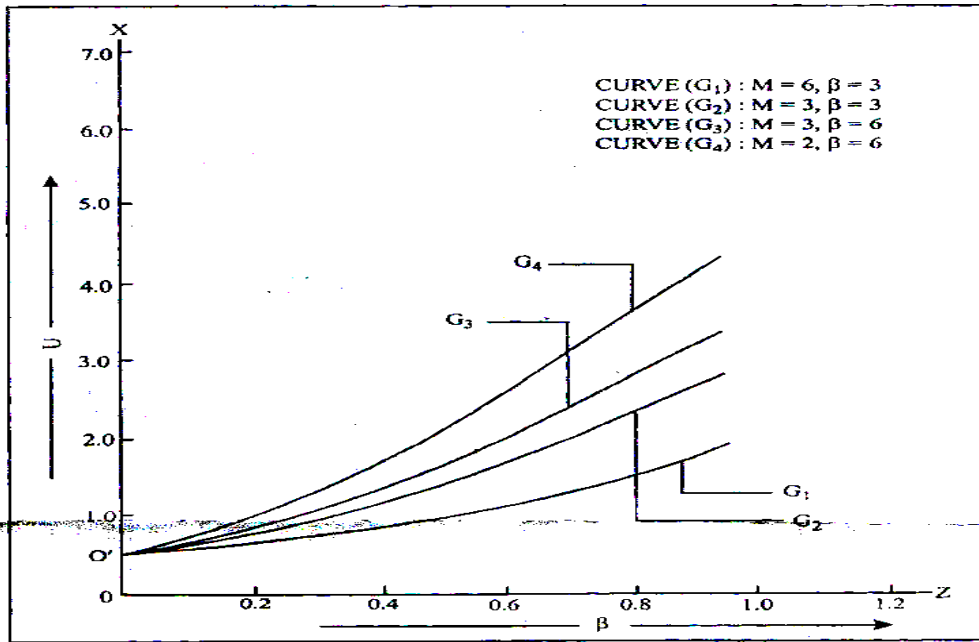


Fig. (1)

we have also,

$$I = [\sin \omega t - e^{(2d-z)F \cos \phi} \sin (Pz \sin \phi - \omega t) + e^{Pd \cos \phi} \sin (Pd \sin \phi + \omega t) - e^{P(d-z) \cos \phi} \sin \{P(d-z) \sin \phi + \omega t\}] + [1 + e^{2dP \cos \phi} - e^{Pd \cos \phi} \cos (Pd \sin \phi)] \dots (3.26)$$

where the values of  $P$  and  $\phi$  are given by,

$$P = \sqrt{16\omega^2 + (\beta^2 + 4M^2)^{1/2}} \dots (3.27)$$

and

$$\phi = \frac{1}{2} \tan^{-1} \frac{4\omega}{\beta^2 + 4M^2} \dots (3.28)$$