PAIRING METHODS OF VARIABLES BASED ON THE RELATIVE GAIN ARRAY AND ITS EXTENSIONS APPLIED TO A COLUMN SEPARATION

Licianne Pimentel Santa Rosa, Cássio Henrique Andrade, Roberta Flávia Romero, Laio Damasceno da Silva.

Abstract — **Modern chemical plants are composed by several elementary units and unit operations cross-linked through matter recycle and energetic integration. These linkages are very complex and have many variables that can be manipulated and controlled. So, they spread a wide variety of possible control meshes and need some control perspectives that are not bounded to individual units, but to the wide plant. In this context, the objective of this paper is to presented pairing methods of variables based on the relative gain array and its extensions and applied them to a column separation. The extensions of the RGA method evaluated here are DRGA, PRGA, RNGA and ERGA. These methods were implemented on Matlab® and the results show that each extension provides the same pairing. The ideal pairing was the one in which determined pair ui-yi appeared more often. So, this paper motivates future studies especially regard to application of the paring methods to a complex plant with a lot of unit's operations.**

Keywords: gain array; pairing methods; separation column.

I. INTRODUCTION

Modern chemical plants are made up of several elementary units and unit operations that are increasingly interconnected through the recycling of matter and energy integration. They are therefore extremely complex and have numerous variables that can be manipulated and controlled, thus opening up a range of possible control loops and therefore requiring control perceptive that are not limited to individual units, but rather the plant as a whole. The design of the control structure must be carried out in such a way as to choose the best variables for measurement, manipulation and control. Another important point in this project is the definition of how these variables should be interconnected to ensure good performance and controllability of the plant.

The purpose of the Plantwide control project is to promote a global control structure for many important variables in multi-unit processes, including the entire plant's output and product quality. The hierarchical procedure consists of first clearly defining the process objectives and identifying the operational constraints. Subsequently, the controlled and manipulated variables must be selected and their pairing established. Finally, the most appropriate control configuration and the economic classification of the process are established.

With regard to determining the variables manipulated and controlled, the most widely used methods, the RGA (Relative Gain Array), first presented by Bristol (1966), is one of the most relevant methods for breaking down MIMO (multi-input-multi-output) systems into SISO (single-input-single-output). This tool allows control pairs to be defined, allowing control structures capable of producing less interaction between variables to be visualized. A great advantage of using this tool is that it is independent of scheduling, but there are limitations, and to remedy these, research groups have dedicated their studies to developing extensions to the RGA method.

There are different techniques for selecting manipulated and controlled variables used in the design of controllers by controllability analysis, such as: the Relative Gain Matrix (RGA) method and its extensions on the methods resulting from its extension, such as: DRGA, PRGA, RNGA and ERGA. When designing controllers, the first problem to be solved is the selection of control structures or the question of pairing the input and output variables. Kookos and Lygeros (1998) state that the selection of the control structure can dramatically affect the performance of the controller. The number of alternative control structures is enormous, especially in global plant control problems. For this reason, a lot of work has been done, especially with regard to the development of algorithms that facilitate the selection of the control structure and are based on the evolution of the interaction of the -loop‖ associated with each control structure.

So, this study aims to presented pairing methods of variables based on the relative gain array and its extensions and applied them to a column separation. The extensions of the RGA method evaluated here are DRGA, PRGA, RNGA and ERGA. These methods were implemented on Matlab®. The RGA was introduced by Bristol 1966 from it developed a concept to quantify and evaluate the static interaction between two SISO control loops. According to Hofmann et al. (2019), its extension to an arbitrary finite number of SISO control loops as well as to the dynamic case is described in Tung and T. Edgar (1981), Seborg et al, (2010) and Witcher (2007) is often referred to as DRGA. Additionally, there exist numerous other studies dealing with the application of RGA (Shinskey, 1981, Papadourakis

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et al., 1987 and McAvoy and Yeh, 1994) about multivariable control.

II. METODOLOGY

2.1. Relative Gains Matrix (RGA)

Kookos and Lygeros (1998) point out that RGA has been recognized as an efficient measure of interaction in both academia and industry. Seborg (1989) states that using this method generates two important pieces of information about the process:

- A measure of process interaction;
- An effective recommendation from the pair of controlled and manipulated variables.

This method is based on the concept of relative gains. Considering a process with n controlled variables and n manipulated variables, the relative gain, λij, between the controlled variable and the manipulated variable is defined as the ratio of gains in two states:

$$\lambda_{ij} = \left(\frac{\partial C_i}{\partial M_j}\right)_M \Big/ \left(\frac{\partial C_i}{\partial M_j}\right)_C = \frac{Ganho\ "open-loop"}{Ganho\ "closed-loop"} \quad (1)$$

where the derivative $\left(\frac{\partial C_i}{\partial M_j}\right)_C$ indicates the effect of M_i on C_i when all the other control loops are closed. After determining the gain, it is necessary to obtain the relative gain matrix at zero frequency, as shown in Equation (2).

$$RGA(0) = G \otimes [G^{-1}]^T \quad (2)$$

As the relationship that defines the RGA only makes sense for systems where each manipulated variable controls only one input of the system, we will only study so-called square systems, where the number of manipulated variables is equal to the number of controlled variables, or if we analyze the transfer matrix of the plant model, the number of rows is

$$e_{ij} = g_{ij}(0)\int_0^{\omega_{B,ij}} |g_{ij}^0|\, d\omega \quad (5)$$

equal to the number of columns (Farina, 2000). Equation 3 represents the configuration of a relative gain matrix.

$$RGA = \begin{array}{c} \\ C_1 \\ C_2 \\ \vdots \\ C_n \end{array} \begin{bmatrix} M_1 & M_2 & \cdots & M_n \\ \lambda_{11} & \lambda_{12} & \cdots & \lambda_{1n} \\ \lambda_{21} & \lambda_{22} & \cdots & \lambda_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ \lambda_{n1} & \lambda_{n2} & \cdots & \lambda_{nn} \end{bmatrix} \quad (3)$$

Once the relative gain matrix has been obtained, the pairings are made according to the degree of interaction between the suggested pairs. This tool enables control pairs to be defined, allowing the structures capable of producing less interaction between the variables to be visualized. Two important properties are related to the relative gains matrix: It is normalized when the sum of the elements in each row and column is equal to 1; The relative gains are not affected by the choice of units or the scale of the variables.

2.2. Dynamic Relative Gains Matrix (DRGA)

In order to study these processes dynamically, Witcher and McAvoy (1977) evaluated controllability over a range of frequencies of interest or at a specific frequency within this range. This gave rise to the DRGA, which is the dynamic extension of the RGA method. This dynamic extension of the RGA is represented by Equation 4 and G(s) represents the transfer function matrix of an n x n plant:

DRGA(s)=G(jω)⊗[G^(-1) (jω)]^T (4)

where the symbol ⊗ denotes element-by-element multiplication (Hadamard or S-chur product). This method has the same properties as its steady-state equivalent, except that its elements are calculated over a certain frequency range.

2.3. Dynamic Relative Gains Matrix (DRGA)

Xiong et. al. (2005) developed a modification of the RGA, presenting a new criterion for decentralized control of multivariable processes through a dynamic pairing cycle. Instead of using the steady-state matrix G, they proposed a composite integral frequency matrix of the absolute value of the elements of G. The limits of the integral frequency range from 0 to the broadband of the system. This extension of the RGA method is called ERGA and takes into account the wide range of operating frequencies. By defining an effective gain in the matrix, the pairing cycle procedures are carried out. This extension is based on the popular relative gain matrix method which is directly extended to the new method that can reflect dynamic interactions in the loop under finite bandwidth control. Compared to existing methods, this method is simple, effective and easy for control engineers to understand and apply (Toro, 2008).
The introduction of effective gains takes into account the steady-state gain and response speed information to measure the interaction and loop and perform the pairing, the effective gain e_ij for a given transfer function is now defined as:

Figure 1 shows the response curve and the effective energy of the transfer function $g_{ij}(j\omega)$. Onde, ω_{ij} para i, j = 1,2,... n are the broad bands of the transfer function.

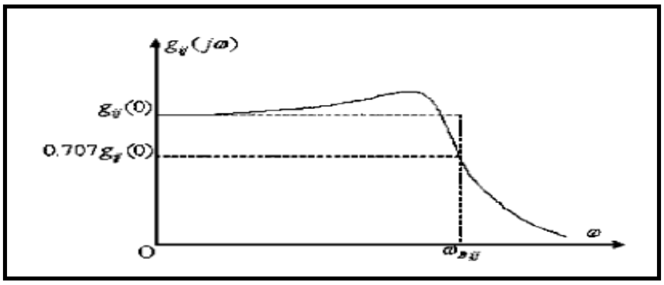

Figure 1: Response curve and effective energy of $g_{ij}(j\omega)$ (Xiong et. al, 2005)

When all the relative effective gains are calculated by $\Phi_{ij} = \frac{e_{ij\,1}}{e_{ij\,2}}$, the relative effective gains matrix is obtained for the multivariable process:

$$ERGA = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \cdots & \Phi_{1n} \\ \Phi_{21} & \Phi_{22} & \cdots & \Phi_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \Phi_{n1} & \Phi_{n2} & \cdots & \Phi_{nn} \end{bmatrix} \quad (6)$$

Similarly, with the RGA:

ERGA=E⊗ 〖(E^(-1))〗 ^T (7)

The ERGA pairing rules are based on rules obtained with manipulated and controlled variables required in decentralized control and are presented by Xiong et. al. (2005): (1) The sum of the elements corresponding to ERGA are closed to 1.0 (2) NI is positive; (3) All pairs of elements are positive and (4) The selected pair must be the one in which the interaction between the controlled and manipulated variable is greatest.

2.4. Performance Relative Gains Matrix (PRGA)

One of the main criticisms against the use of RGA has been its "failure" in systems that have integration, since only the measure of relative gain can indicate that interaction is not a problem, but coupling in a significant way may exist. To

$$k_N = [k_{ij}]_{n\times n} = G(j0) \odot T_{ar} \quad (10)$$

overcome this problem, Hovd and Skogestad (1992) presented a new measure, called RGA performance (PRGA).

The PRGA matrix was originally introduced in the steady

$$NRGA = k_N \otimes (k_N{}^{-1})^T \quad (11)$$

state by Grosdidier (1990) to understand the effect of interactions in decentralized control, where the term $G_{diag}G^{-1}$ represents the PRGA matrix. This is then defined as:

The elements of the PRGA matrix are given by:

At frequencies where feedback is effective, large elements in the PRGA (compared to magnitude 1) mean that the interactions slow down the response and the overall performance of the system will be worse than for individual loops. On the other hand, small PRGA elements (compared to magnitude) mean that the interactions have improved the system's performance at a certain frequency.

2.5.Performance Relative Gains Matrix (PRGA)

This extension of the RGA method, known as the normalized relative gain matrix, proposes gains based on measures of interaction between the variables and was proposed by He et. al. (2009). The principle of this method is to investigate the process in both the steady state and the transient, both using information from the system's transfer functions. The normalized relative gain matrix (NRGA) was introduced for loop interaction measurements. Consequently, a new pairing cycle based on the RNGA criterion is proposed for the configuration of the control structure.

According to He et. al. (2009), the main advantages of this method are:

- Compared to the RGA method, the NRGA considers not only steady-state process information, but also transient information;
- Compared to the DRGA method, it also provides a comprehensive description of the dynamic interaction between the individual loops without requiring the specification of the controller type and with much less computation;
- Compared to the ERGA method, it requires even less computation, but results in a single loop and optimal pairing decision;
- It is very simple for field engineers to understand and work out pairing decisions in practical applications.

Given the definitions of the residence time and the relative gains in frequency, the normalized gain matrix is obtained, as:

where, T_{ar} stands for normalized residence time matrix, and the symbol $\odot$ indicates element-by-element division. To obtain the normalized relative gains, the k_N matrix is multiplied member by member by its inverse transpose:

$$PRGA(s) = G_{diag}G^{-1} \quad (8)$$

$$\text{U}_{ij} = \lambda_{ij}\frac{g_{ii}}{g_{ij}} \quad (9)$$

III. Results and Discussions

The purpose of this section is to apply the methods presented above. For this purpose, a 2 by 2 matrix problem was used. The problem consists of a transfer function model of a conventional distillation column proposed by Chiang and Luyben (1998). The transfer function model of a conventional distillation column proposed as shown:

$$\begin{bmatrix} x_D \\ x_B \end{bmatrix} = \begin{bmatrix} \frac{3,6}{(12s+1)(4s+1)} & \frac{-4,44}{(15,5s+1)(2s+1)} \\ \frac{12,2e^{-s}}{L\,(19s+1)(s+1)} & \frac{-33,4}{(23s+1)(s+1)} \end{bmatrix} \begin{bmatrix} R \\ Q_R \end{bmatrix} \qquad (12)$$

where xD and xB represent the partial fractions of the components in the distillate and at the bottom of the column, R and QR represent the reflux flow rate and heat input, respectively, as Figure 2 despites.

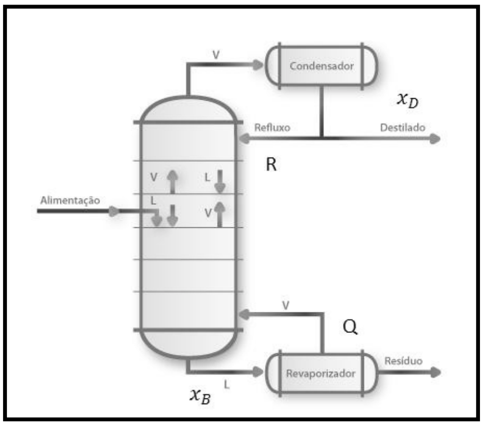

Figure 2: **S**chematic representation of a distillation column.

First, the RGA method was applied and the relative gains matrix was obtained, as shown in the matrix below.

Therefore, the RGA suggests that the concentration of species D is controlled by the reflux rate and that the concentration of species B is controlled by the amount of heat added to the system. After the pairing recommended by the RGA at zero frequency, the extensions of this method were used, as explained above. The first extension to which the case study was applied was the dynamic RGA.

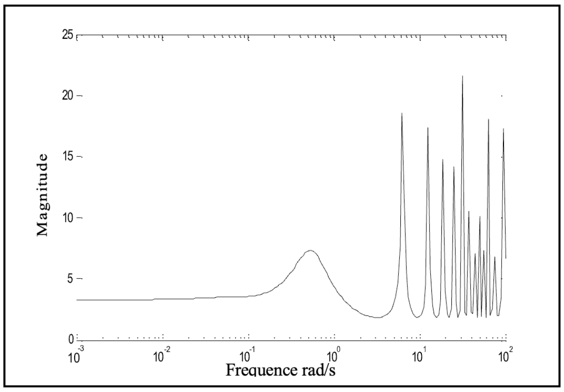

Figure 3: RGA Number

As can be seen in Figure 3, the RGA number is small for low frequency values, so in this case there is little interaction in the system (or it is negligible) and the diagonal control structure is adequate, which implies stability of the individual loops and therefore global stability of the system. The cut-off frequency was then determined. Figure 4 shows that the end of the open-loop bandwidth occurs at frequency $\omega \approx 1.4$ rad/sec, which is given by the intersection of the maximum singular value by the -3 dB line.

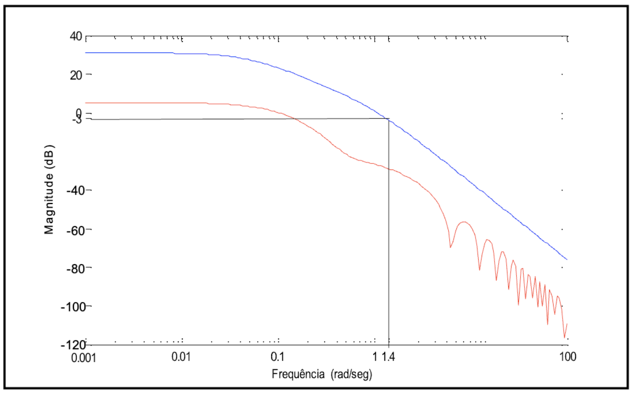

Figure 4: Unique maximum and minimum value

After obtaining the cut-off frequency, the relative gain matrix was applied to that frequency. Therefore:

$$RGA(w_c) = \begin{bmatrix} \mathbf{5.1402} & -4.1402 \\ -4.1402 & \mathbf{5.1402} \end{bmatrix} \qquad (14)$$

Equation 14 shows that the proposed pairing was the same as the RGA at zero frequency. The predominance of the diagonal structure throughout the frequency range of interest can be verified using

$$RGA = \begin{bmatrix} \mathbf{1,8198} & -0,8198 \\ -0,8198 & \mathbf{1,8198} \end{bmatrix} \qquad (13)$$

Equation 6, proposed by Skogestad and Postlethwaite (1996).

The relative gain performance method was then applied to the distillation column and the following matrix was obtained:

$$PRGA = \begin{bmatrix} \mathbf{1,8198} & -0.2419 \\ -6.1672 & \mathbf{1,8198} \end{bmatrix} \qquad (15)$$

It is important to note that for this case the system did not need to be normalized, since the aim of this section is only to apply the methods and not to design controllers. Suggesting a diagonal pairing, where the concentration of D is controlled with the reflux flow rate and the concentration of B with the amount of heat in the system.

Subsequently, the normalized relative gain method was used, where dynamic system information was taken into account:

$$NRGA = \begin{bmatrix} \mathbf{1,9773} & -0.9773 \\ -0,9773 & \mathbf{1,9773} \end{bmatrix} \qquad (16)$$

It should be noted that the recommended pairing is the same as the RGA at zero frequency, i.e. xD-R and xB-Q. Finally, the effective relative gain method was applied and the following matrix was obtained, which represents the frequencies recommended by the method:

$$\Omega = \begin{bmatrix} 0{,}07151 & 0{,}0646 \\ 0{,}0511 & 0{,}0425 \end{bmatrix} \quad (17)$$

Now multiply Equation 17 and Equation 16 to obtain the effective gain. This gives us:

$$E = \begin{bmatrix} 0.1367 & -0{,}0522 \\ -0{,}0419 & 0{,}0773 \end{bmatrix} \quad (18)$$

Finally, the definition of the effective relative gains gives a matrix:

$$ERGA = \begin{bmatrix} \mathbf{1{,}2657} & -0{,}2657 \\ -0{,}2657 & \mathbf{1{,}2657} \end{bmatrix} \quad (19)$$

In view of this, it can be seen that the recommended pairing, as with all the other methods, is the diagonal one with the following combination: xD-R and xB-Q. Table 2 summarizes these results.

Table 2 - Summary of case study results

Method	Recommended pairing	Pair of variables
RGA	$\begin{bmatrix} \mathbf{1{,}8198} & -0{,}8198 \\ -0{,}8198 & \mathbf{1{,}8198} \end{bmatrix}$	x_D-R e x_B-Q
DRGA	$\begin{bmatrix} \mathbf{5.1402} & -4.1402 \\ -4.1402 & \mathbf{5.1402} \end{bmatrix}$	x_D-R e x_B-Q
PRGA	$\begin{bmatrix} \mathbf{1{,}8198} & -0.2419 \\ -6.1672 & \mathbf{1{,}8198} \end{bmatrix}$	x_D-R e x_B-Q
NRGA	$\begin{bmatrix} \mathbf{1{,}9773} & -0.9773 \\ -0{,}9773 & \mathbf{1{,}9773} \end{bmatrix}$	x_D-R e x_B-Q
ERGA	$\begin{bmatrix} \mathbf{1{,}2657} & -0{,}2657 \\ -0{,}2657 & \mathbf{1{,}2657} \end{bmatrix}$	x_D-R e x_B-Q

CONCLUSION

In this study, a systematic approach to the tools used to pair variables was presented. was presented. As an illustration, a classic separating column problem was presented. Matlab implementations of the methods for obtaining relative gain matrices (RGA, DRGA, NRGA, PRGA and ERGA) were carried out. The analyses of the variable pairing tools show that each method provides equal pairs, but it is clear that some gains are constant in the different methods, such as RGA and PRGA. These results motivate future studies, especially with regard to the application of these methods to global plant control, since the focus here was only on presenting them systematically and applying them to a very simple piece of equipment.

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