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Self-organizing Learning Dynamics in Population-based Meta-heuristic Optimization

Jiann-Horng Lin, Rong Jeng

Abstract— **We consider the learning processes in collective dynamics of optimization search. The self-organizing behavior in meta-heuristic learning and the role of entropy are presented. A quantitative measure of the degree of self-organization as a function of coupling parameter is given through information measures. Our assumption in this paper is that an abstract and generic model exists that unifies some population-based meta-heuristics. The model system described here is a coupled map lattice which serves as a paradigm for the spatiotemporal behavior of coupled nonlinear systems. The self-organization processes are investigated in the framework of coupled learning dynamics, which also governs the mechanisms underlying the learning processes. This paper is mainly devoted to aspects of self-organization and evolution associated with these concepts. We have presented a way to analyze the mechanism of collective dynamics on the basis of the spatial KS entropy for the measurement of the transition from spatially disordered to ordered behavior.**

Index Terms— **Meta-heuristic Optimization, Information Theory, Learning Dynamics, Self-organization**

I. INTRODUCTION

Systems are usually called complex when they have great number of elements that interact among each other in a complicated fashion, and among these elements there are probabilistic relations. The processes of self-organization take place by the transformation of the existing relations and the rise of new relations between the elements of the system. Self-organization is characterized by the process in which the organization of complex systems is being created, reproduced or improved. Self-organization is conditioned by the effects of nonlinearity and dissipation. The onset of structures in the self-organization processes is a specific type of evolution: the evolution from disorder to order. Like symmetry and asymmetry, the concepts of disorder and order complement each other, there is no sharp distinction between them. The self-organizing processes can be considered as a form of transition into ordered states in complex dynamics. According to the Second Law of Thermodynamics, in equilibrium, any isolated system tends to achieve a state of maximum disorganization. Entropy is quantitative measure of disorganization. Following the traditional interpretation of entropy, it is disorder that is taken to be measured by entropy. The identification of entropy with disorder is related to the concept of information as a decrease in entropy. A higher value of entropy results from a more even distribution for the probability density. Entropy is evidently a good measure for disorder, and consequently some sort of inverse of entropy should be a measure for order. The entropy decreases while the system becomes organized. In this way the conclusion was derived that entropy is an appropriate measure for the description of self-organization. On the other hand, synchronization is a key concept to the understanding of self-organization phenomena occurring in the fields of two coupled oscillators. The self-organizing principle underlying the collective behavior of coupled oscillators is that of synchronization or mutual entrainment. As a result of this phenomenon the interacting subsystems demonstrate the tendency to oscillations with equal or rationally related frequencies. Natural systems are often constituted by several nonlinear units connected in complex topologies. It can be observed how natural complex systems are intrinsically adaptive and cooperative. In particular, synchronization emerges as one of the main issues concerning adaptation and cooperation. Coupled oscillators are spatiotemporal dynamical systems. Large assemblies of oscillating elements can spontaneously evolve to a collective organization even if each element has a complex dynamical behavior. In order to stress the specific role of cooperative, collective effects in the processes of self-organization, we consider global synchronization in lattice dynamical systems and propose a measure of the self-organizing process in this paper.

Jiann-Horng Lin and Rong Jeng, Dept. of Information Management, I-Shou University, Taiwan

II. GLOBALLY COUPLED LEARNING DYNAMICS

The study of complex dynamical behavior in spatially extended systems is of interest in a wide variety of contexts. Spatiotemporal structures can arise when large numbers of dynamical elements are coupled. As discrete analogs to coupled oscillators and partial differential equations, coupled map lattice have been attracted much attention in the study of spatiotemporal chaos and pattern formation as models of spatially extended systems. Coupled map lattice (CML) systems, first introduced by Kaneko [1] are simple and popular models for studying spatial-temporal behavior of systems and seem to be gaining popularity as tools for modeling complex phenomena in physics, engineering, biology, chemistry, social sciences, economics, etc. A coupled map lattice is an *N*-dimensional dynamical system of interconnected units where each unit evolves in time through a map or recurrence equation of the discrete form

$$X^{k+1} = F(X^k) \tag{1}$$

where X^k denotes the field value (*N*-dimensional vector) at the indicated time *k*. In the case of a globally coupled map, with a global coupling factor ε, the dynamics can be rewritten as

$$x_n^{k+1} = (1-\varepsilon) f_n(x_n^k) + \frac{\varepsilon}{L}\sum_{j=1}^{L} f_j(x_j^k) \tag{2}$$

where *n* and *j* are the labels of lattice sites ($j \neq n$). The term *L* indicates over how many neighbors we are averaging. The local dynamics of coupled map lattice is giving by a nonlinear map and a coupling factor. Kaneko proposes globally coupled map (GCM) systems. GCM systems consist of chaotic elements that are globally coupled. He has investigated their dynamic behavior and information processing ability thoroughly [2]. The spatially extended nature of the system permits the appearance of complex spatiotemporal behavior. Though CML models are idealized systems, they are sufficiently complex to be capable of capturing the essential features of the dynamics of the system, and at the same time have the advantage of being mathematically tractable and computationally efficient. Due to the large number of degrees of freedom in such spatially extended systems, a variety of spatiotemporal phenomena, like synchronization, intermittency, and spatiotemporal chaos, are observed [1]. One of the most important and interesting modes which can arise in such systems is the mode corresponding to synchronized behavior, i.e., behavior in which evolution at each spatial location is identical with that at every other spatial location at any arbitrary instant of time. In this section we focus our attention on the interesting and widely observed phenomenon of synchronization, i.e., spatially homogeneous behavior in coupled systems and study this phenomenon in the context of synchronization in a coupled map lattice.

One of the most studied versions of CML is the nearest-neighbor coupling case in one space dimension which can be given by

$$x_{n+1}(i) = \alpha f(x_n(i)) + \beta g[f(x_n(i)), f(x_n(i+1)), f(x_n(i-1))]$$

where *n* is a discrete time step and *i* is a lattice point (*i= 1, 2,…, N* = system size).

The function *f(x)* is a nonlinear mapping and the function *g* can, for instance, be chosen as the diffusive coupling $g = f(x_n(i+1)) + f(x_n(i-1)) - 2f(x_n(i))$, one-way coupling $g = f(x_n(i)) + f(x_n(i-1))$, or models with global coupling, $g = \sum_{j=1}^{N} f(x_n(j))$, and so on.

A typical example is the following diffusively coupled model [9]:

$$x_{n+1}(i) = (1-\varepsilon) f(x_n(i)) + \frac{\varepsilon}{2}[f(x_n(i+1)) + f(x_n(i-1))] \quad (3)$$

The model has been investigated as a prototype for chaos in spatially extended systems, including extensions to a high-dimensional lattice, a different choice of nonlinear function *f(x),* and different types of couplings. Note that the equivalent dynamics to equation (1) is obtained by the transformation $y_n(i) = f(x_n(i))$,

$$y_{n+1}(i) = f((1-\varepsilon) y_n(i) + \frac{\varepsilon}{2}[y_n(i+1)) + y_n(i-1)]) \quad (4)$$

This form may be more familiar with researchers in artificial neural networks, if one chooses a sigmoid function (e.g., $\tan(\beta x)$) as *f(x)* and the coupling term ε depending on elements. Such extended nonlinear dynamical systems are capable of irregular behavior such as spatiotemporal chaos as well as a variety of ordered and regular behavior in space and time. They demonstrate a rich variety of self-oscillating regimes from simple regular to complex ones. This challenges to introduce a quantitative complexity criterion that allows evaluating the degree of order in such different regimes.

Contrarily, we consider some population-based meta-heuristic optimization search models as one class of GCM systems.

Particle swarm optimization (PSO) is a population-based evolutionary computation technique, and was originally developed by Kennedy and Eberhart [3]. It is inspired by social behavior among individuals. These individuals, called particles, are moving through an n-dimensional search space, each particle represents a possible solution of the problem, and can remember the best position (so1ution) which they have reached. All the particles can share their information about the search space, so there is a global best solution. Bat-inspired algorithm is an another meta-heuristic optimization algorithm developed by Xin-She Yang [4]. This bat algorithm is based on the echolocation behavior of microbats with varying pulse rates of emission and loudness. Furthermore, the firefly algorithm [5] is also a meta-heuristic algorithm, inspired by the flashing behavior of fireflies. The primary purpose for a firefly's flash is to act as a signal system to attract other fireflies. In firefly algorithm, the flashing light can be formulated in such a way that it is associated with the objective function to be optimized, which makes it possible to formulate the firefly algorithm. Many other algorithms have been devised to improve its performances for the optimization problems. Furthermore, to enrich the searching behavior and to avoid being trapped into local optimum, some meta-heuristic search algorithms intended to introduce chaotic dynamics and Levy flights into the algorithm are presented in our paper [6]. We proposed some synergistic approaches to meta-heuristic search optimization algorithms. Some new approaches to bat algorithm and firefly algorithm as the synergistic meta-heuristics are developed.

Generally, the dynamics of move generations in these optimization search algorithms can all be rewritten as

$$X^{k+1} = G(X^k) \quad (5)$$

and can enable the analysis of their convergence behavior through GCM. The meaning of these move generation models as the GCM lies in the variety of maps *G*. The move generations of all these population-based meta-heuristic models are recognized to be one class of GCM systems. The characteristic of the class is that the local solution is transformed by a nonlinear map and connected to other solutions through the control parameter of the map. That is, in these models, the move generation that is a nonlinear transformation from $x_i(n)$ to $x_i(n+1)$ is decided by $x_j(n)$ ($j \neq i$), at each discrete time *n*. The model structure we have just described is abstract and generic. This enables to make clear the involved mechanisms in the learning dynamics.

III. A MEASURE OF SELF-ORGANIZING DYNAMICS

The self-organizing learning processes in the collective dynamics of some population-based meta-heuristic optimization search described in Section Ⅱ are coupled map lattices which serve as a paradigm for the spatiotemporal behavior of coupled nonlinear systems. A natural parameter for studying the self-organization of a system is the entropy. However, many forms of entropy have been identified: Information entropy, Kolmogorov entropy, etc. How practical are these parameters in determining whether or not a system is spontaneously organizing? Mutual information provides a measure of the quantitative changes in the synchronization of two coupled chaotic systems. In this section, we propose to use the spatial Kolmogorov-Sinai (KS) entropy to quantify the degree of self-organization in lattice dynamical systems. In order to formulate a definition of the spatial KS entropy in spatially extended systems, the subspace Lyapunov spectrum and subspace KS entropy are defined as the following.

A. Subspace Lyapunov spectrum and Subspace Kolmogorov-Sinai entropy

Lyapunov spectra [7] characterize how a small disturbance in tangential space is amplified or contracted. In an *N*-dimensional dynamical system, there are *N* independent tangential vectors. Corresponding to them there exist *N* eigenvalues, which form the spectrum. In CML, Lyapunov spectra can be defined by the product of Jacobi matrices. The logarithms of the eigenvalues of the product, divided by time steps n with the limit $n \to \infty$ give Lyapunov exponents [8]. The exponents λ_i ordered from the largest to the smallest, give a spectrum. Sum of the positive Lyapunov exponents give the amplification ratio of an *N*-dimensional tangential volume. It is equal to KS entropy which quantifies the mean rate of information production in a system, or alternatively the mean rate of growth of uncertainty in a system subjected to small perturbations. We take a N_S-dimensional subsystem S_{N_s} starting at any position *j*, subspace consisting of sites *j*, *j +1,…, j+* N_S *-1*, and define Lyapunov spectra at the subspace S_{N_s}. The calculation of these subsystem exponents $\lambda_i^{(S)}$ is in the same manner as usual Lyapunov exponents of the full system. That is, given a *N*-dimensional dynamical system defined by a map $f : R^N \to R^N$, the subspace Lyapunov exponents associated to the subspaces $R^{N_S} \times R^{N-N_S}$ are defined by the logarithms of the eigenvalues of the matrix

$$\Lambda_{N_S} = \lim_{n\to\infty}([D_{N_S} f^n(x)]^T [D_{N_S} f^n(x)])^{1/2n} \qquad (6)$$

where $D_{N_S} f^n$ is the $N_S \times N_S$ diagonal block of the full Jacobian. Here the amplification/contraction of small disturbances at the boundaries ($x(j-1)$ and $x(j+N_S)$) is neglected. Boundary effect comes in only through the motion of $x(j)$ and $x(j+N_S-1)$.

From Pesin's formula, the sum of all positive Lyapunov exponents provides an estimate of KS entropy. Therefore, subspace KS entropy $h_{KS}^{(S)}$ is straight-forwardly defined by replacing Lyapunov spectra in equation (6) by the subspace Lyapunov spectra,

$$h_{KS}^{(S)} = \sum_{\lambda_i^{(S)}>0} \lambda_i^{(S)} \qquad (7)$$

B. Spatial Entropy Measurements of Collective Dynamics

The Kolmogorov-Sinai entropy of a dynamical system measures the rate of information production per unit time. That is, it gives the amount of randomness in the system that is not explained by the defining equations. Hence, the subspace KS entropies may be interpreted as a measure of the randomness that would be present if the two subsystems S_{N_S} and S_{N-N_S} were uncoupled. The difference $h_{KS}^{(N_S)} + h_{KS}^{(N-N_S)} - h_{KS}$ represents the effect of the coupling. For a *N*-dimensional dynamical system *S* with two subsystems S_{N_S} and S_{N-N_S} , we define the *spatial entropy* $\Omega(S)$ for a measure of self-organizing dynamics as

$$\Omega(S) = \frac{1}{N}\sum_{N_S=1}^{N} h_{KS}^{(N_S)} + h_{KS}^{(N-N_S)} - h_{KS} \qquad (8)$$

where h_{KS} and $h_{KS}^{(N_S)}$ are the KS entropies for the system *S* and subsystem S_{N_S}, respectively. $h_{KS}^{(N-N_S)}$ denotes the subspace KS entropy of the complement of the subsystem S_{N_S}. On the basis of the Lyapunov spectra, the KS entropies can be estimated. The spatial entropy is similar to mutual information for a quantitative measure of the synchronization process between two chaotic systems. $\Omega(S)$ provide a way of quantifying transition from disordered to ordered behavior in spatially extended systems.

IV. Preliminary Simulation Results – CML as an Example

Similar to the mutual information analysis of the synchronization of two coupled nonlinear dynamical systems, we propose a quantitative measure of self-organizing dynamics for the transition from spatiotemporal nonlinear dynamics to synchronized nonlinear dynamics in complex systems as described in the previous section. As an illustrative example of the application of spatial KS entropy in a simple system, we would like to demonstrate the spatial KS entropy behavior during self-organizing processes in a lattice dynamical system of *N* coupled, one-dimensional maps. The lattice evolution is described by

$$x_{n+1}(i) = (1-\varepsilon) f(x_n(i)) + \frac{\varepsilon}{2}[f(x_n(i+1)) + f(x_n(i-1))]$$

where $x_n(i)$ is the variable associated with the *i*th lattice at time *n* taking values in a suitably bounded phase space. *f(x)* is the function describing the local dynamics, for which $f(x) = 2x(\text{mod}1)$. This map has the property that any initial condition on the interval [0, 1] will remain on that interval under the action of the map. It is a chaotic dynamical system with positive KS entropy equal to ln 2 [9].

In Figure 1 (a)-(d), scaled Lyapunov spectra are plotted with parameter values ε = 0.25, 0.45, 0.55 and 0.75, respectively. For large set of model parameters, spatiotemporal chaos is found. These spatially extended systems exhibit chaotic motion with a large number of positive Lyapunov exponents. Figure 2 shows the spatial entropy measurement $\Omega(S)$ as a function of the coupling parameter ε for *N = 100*. Starting from random initial conditions, at ε = 0.25 and ε = 0.45, the evolutions of the system are disorganized (Figure 3 (a) and (b)). Above $\varepsilon \approx 0.5$, the system becomes fully synchronized (Figure 3 (c) and (d) for ε = 0.55 and ε = 0.75, respectively), after a short transition period. At this range, the system shows an organizational structure. Systems that display this behavior are temporally chaotic, but spatially ordered or coherent. Here the coherence is of a particular type - the dynamics is the same or nearly so for long periods of time for all coupled subsystems or large regions of them. Figure 3 shows the overlapped time series of $x_n(i)$ for the first 200 time steps. The space-time behavior with different parameter values is illustrated in Figure 4. These transitions result in more ordered state and the decrease of entropy which points to self-organization. We find that there is a range of coupling strength for which synchronized nonlinear dynamics exists. Outside that range, synchronization breaks down and the system enters a regime of spatiotemporal nonlinear dynamics. The loss of synchronization is accompanied by spatially disordered behavior.

These figures shows that synchronized nonlinear dynamical time series can be generated spontaneously in a spatially distributed system. Synchronization is possible only within a range of coupling strength. When synchronization breaks down we observe spatiotemporal nonlinear dynamics. These results may have relevance in other areas of science where coupled nonlinear systems are used to model self-organization and spatiotemporal complexity.

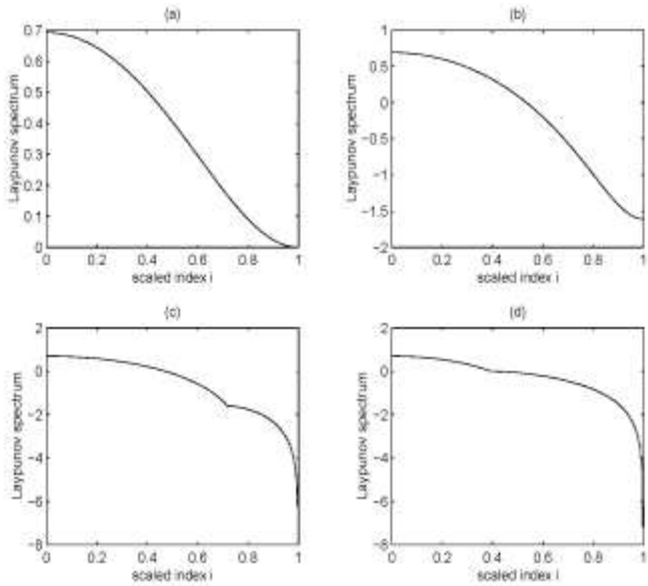

Fig.1. Scaled Lyapunov spectra for coupled map lattices, with (a) $\varepsilon = 0.25$ (b) $\varepsilon = 0.45$ (c) $\varepsilon = 0.55$ (d) $\varepsilon = 0.75$

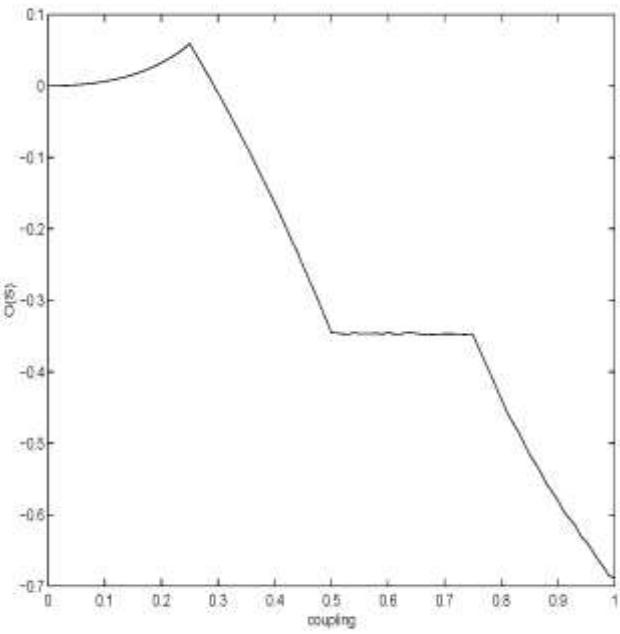

Fig.2. Spatial entropy measurement of self-organizing dynamics for a CML

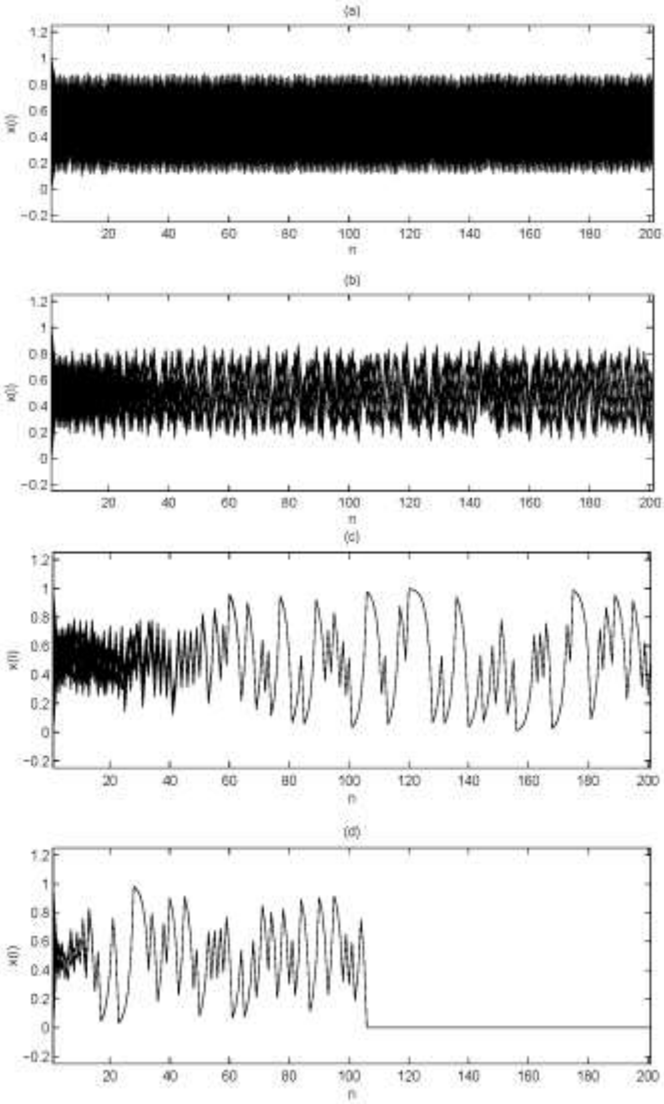

Fig.3. The overlaid time series of $x_n(i)$, $i = 1,...,100$, with (a) $\varepsilon = 0.25$ (b) $\varepsilon = 0.45$ (c) $\varepsilon = 0.55$ (d) $\varepsilon = 0.75$

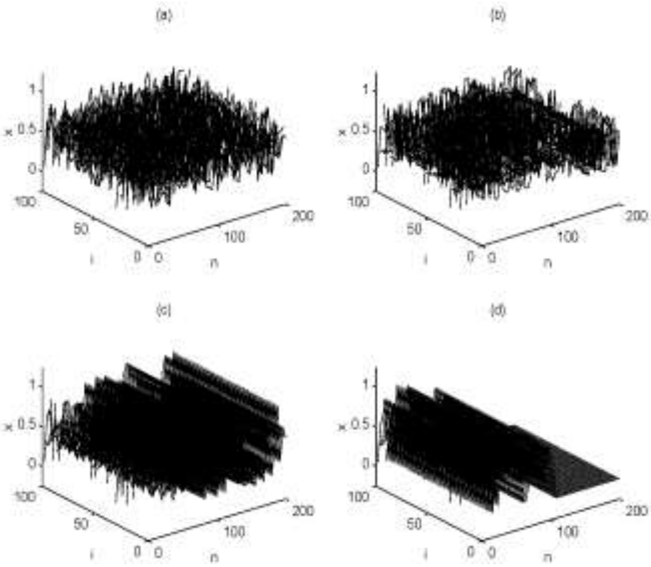

Fig.4. Spatiotemporal evolution of the coupled map lattice, with (a) $\varepsilon = 0.25$ (b) $\varepsilon = 0.45$ (c) $\varepsilon = 0.55$ (d) $\varepsilon = 0.75$

V. Conclusion

From a consideration of population-based meta-heuristic optimization search model as a globally coupled map, we investigated the self-organizing learning dynamics. In this paper, entropy is used for the analysis of complex dynamical behavior in the spatially extended systems. The characterization of spatiotemporal behavior in such systems can provide insights into the complex behavior found in diverse systems. We have tested it on some small instances of the coupled map lattice with promising results. When organization does not take place, the most interesting phenomenon is the spatiotemporal nonlinear dynamics, in which nonlinear dynamical trends appear both in time and in space. We have presented a way to analyze the mechanism of self-organization on the basis of the spatial KS entropy for the measurement of the transition from spatially disordered to ordered behavior. We think that insights gained from investigation into self-organizing learning dynamics in population-based meta-heuristic optimization will help in formulating similar coupled map lattice idea in more complex systems.

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Author

Jiann-Horng Lin received his B. S. and M. S. both in Computer Science and Information Engineering from Feng-Chia University, Taiwan in 1987 and 1989, respectively. He then received his Ph.D. in Electrical Engineering and Computer Science from Syracuse University, New York in 1999. He is currently an assistant professor at the Department of Information Management at I-Shou University, Taiwan. He is also the department chair from 2004 to 2007. His research interests include artificial intelligence, data mining, chaos and information theory. He is also interested in the area of evolutionary computation and bioinformatics. Dr. Lin is a member of the IEEE, the Association for Computing Machinery and the Institute of Information and Computing Machinery.

Rong Jeng received his B. S. degree in Control Engineering from National Chiao-Tung University, Taiwan in 1981. He then received his M.S. degree in Electrical Engineering from Memphis State University, Memphis, TN in 1986. He received his Ph. D. in Electrical and Computer Engineering from University of Cincinnati, Cincinnati OH in 1991. He is currently an associate professor at the Department of Information Management at I-Shou University, Taiwan. He was also the department chair from 1991 to 1994. His research interests include Operations Research, Simulation, information theory.