Influence of radiation on MHD free convective flow of a Williamson fluid in a vertical channel

Mrs. B.Swaroopa, Prof. K. Ramakrishna Prasad

Abstract— **In this paper, we studied the effects of MHD and radiation on fully developed free convection flow of a Williamson fluid in a vertical channel. The governing non-linear equations are solved for the velocity field and temperature field using the perturbation technique. The effects of various emerging parameters on the velocity field, temperature field, skin friction and Nusselt number are studied through graphs and tables in detail.**

Index Terms— **Williamson fluid, MHD, Nusselt number, radiative heat flux, Hartmann number**

I. INTRODUCTION

The insufficiency of the classical Navier-Stokes theory to describe rheologically complex fluids such as blood, paints and polymeric solutions, has led to the development of several theories of non-Newtonian fluids. Non-Newtonian fluid flows are encountered in a wide range of engineering applications. Hot rolling, extrusion of plastics, flow in journal bearings, lubrication, and flow in a shock absorber are some typical examples. The increase of these applications in the past few decades has urged scientists and engineers to provide mathematical models for non-Newtonian fluids. The nonlinearity between stress and deformation rate for this kind of fluids makes it, in general, impossible to obtain a simple mathematical model as in the case for Newtonian fluids. This difficulty has lead researchers to investigate relatively simple non-Newtonian fluid models. A detailed discussion on second and third-order fluids can be found in the study of Dunn and Rajagopal (1995). Hady and Gorla (1998) have investigated the effect of uniform suction or injection on flow and heat transfer from a continuous surface in a parallel free stream of viscoelastic second order fluid. The flow of third grade fluid between heated parallel plates was studied by Akyıldız (2001). Chamka et al. (2002) have discussed the fully developed free connective flow of micropolar fluid between two vertical parallel plates analytically. Siddiqui et al. (2010) have analyzed the flow of a third grade non-Newtonian fluid between two parallel plates separated by a finite gap by using the Adomian decomposition method. Williamson fluid is characterized as a non-Newtonian fluid with shear thinning property, i.e., viscosity decreases with increasing rate of shear stress (Dapra and Scarpi, 2007).

The study of free convective flow in a channel of an electrically conducting viscous fluid under the action of a transversely applied magnetic field has immediate applications in many devices such as magnetohydrodynamic (MHD) power generators, MHD pumps, accelerators, aerodynamics heating, electrostatic precipitation, polymer technology, petroleum industry, purification of crude oil and fluid droplets sprays. Attia and Kotb (1996) have studied the MHD flow between two parallel plates with heat transfer. Free convective flow of micropolar fluid between two parallel porous vertical plates with the effect of magnetic field was investigated by Bhargava et al. (2003). Hayat et al. (2004) have analyzed the Hall effects on the unsteady hydromagnetic oscillatory flow of a second grade fluid. The unsteady flow of a dusty conducting fluid between parallel porous plates was studied by Hazeem attia (2005). Sanyal and Adhikari (2006) have investigated the effects of radiation on MHD fluid flow in vertical channel. The transient MHD couette flow of a Casson fluid between parallel plates with heat transfer was discussed by Attia and Ahmed (2010). Arunakumari et al. (2012) have investigated the fully developed free convection flow of a Williamson fluid in a vertical channel under the effect of magnetic field.

In view of these, we studied the effects of MHD and radiation on fully developed free convection flow of a Williamson fluid in a vertical channel. The governing non-linear equations are solved for the velocity field and temperature field using the perturbation technique. The effects of various emerging parameters on the velocity field, temperature field, skin friction and Nusselt number are studied through graphs and tables in detail.

II. MATHEMATICAL FORMULATION

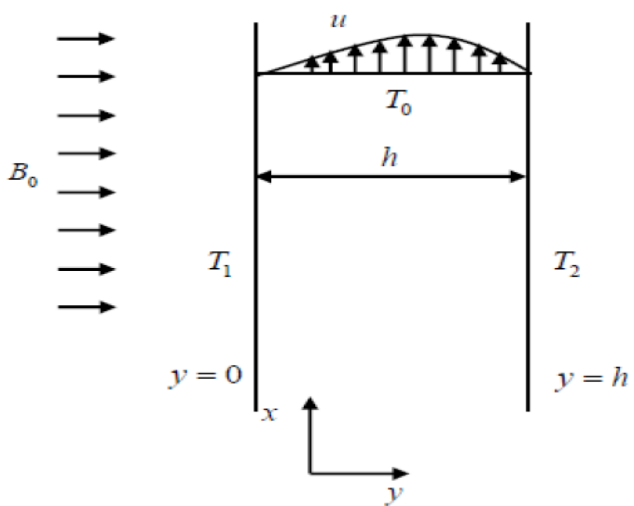

Fig. 1 The physical model

We consider the laminar free convection flow of a Williamson fluid between two plates at distance h a apart with radiative heat transfer, as shown in Fig.1.We choose co-ordinates system, with X - axis parallel to the flow while Y - axis is normal to the flow. A uniform magnetic field B_0

Mrs.B.Swaroopa, Lecturer in Mathematics,Govt. Degree College(W), Srikalahasti Chittoor (Dist), Andhrapradesh – 517644

Prof. K. Ramakrishna Prasad, UGC EMERITUS FELLOW, (Rtd.) Principal & Professor of Mathematics, S.V.University, Tirupati – 517502 , A.P.

is applied perpendicular to the fluid flow. The flow is assumed steady and fully developed, i.e., the transverse velocity is zero. It is also assumed that the walls are heated uniformly but their temperatures may be different resulting in asymmetric heating situation under these assumptions the equations that describe the physical situation are

$$\eta_0 \frac{d^2u}{dy^2} + \eta_0 \Gamma \frac{d}{dy}\left[\left(\frac{du}{dy}\right)^2\right] - \sigma B_0^2 u + \rho g \beta (T - T_0) = 0 \quad (1)$$

$$\frac{k}{c_p}\frac{\partial^2 T}{\partial y^2} - \frac{1}{c_p}\frac{\partial q}{\partial y} = 0 \quad (2)$$

where ρ is the density, g is the acceleration due to gravity, β coefficient of thermal expansion, Γ is the viscoelastic parameter, σ is the electrical conductivity , c_p is the specific heat at constant pressure, k is the thermal conductivity and q is the radiative heat flux. Following Cogley et al. (1968), it is assumed that the fluid is optically thin with a relatively low density and the radiative heat flux is given by

$$\frac{\partial q}{\partial y} = 4\alpha^2 (T_0 - T) \quad (3)$$

Equations here α is the mean radiation absorption coefficient.

Subject to the boundary conditions

$$u(0) = 0, \quad T(0) = T_1, \qquad u(h) = 0, \quad T(h) = T_2 \quad (4)$$

Introducing the following non-dimensional variables

$$\bar{u} = \frac{u}{U}, \bar{y} = \frac{y}{h}, \bar{x} = \frac{x}{h}, We = \frac{U\Gamma}{h}, \theta = \frac{T - T_0}{T_2 - T_0}, r_T = \frac{T_1 - T_0}{T_2 - T_0} \quad (5)$$

into Eqs. (1) and (2), we get (after dropping the bars)

$$\frac{d^2u}{dy^2} + We\frac{d}{dy}\left[\left(\frac{du}{dy}\right)^2\right] - M^2 u + \frac{Gr}{\text{Re}}\theta = 0 \quad (6)$$

$$\frac{d^2\theta}{dy^2} + N^2\theta = 0 \quad (7)$$

where $M = B_0 h\sqrt{\frac{\sigma}{\eta_0}}$ is the Hartmann number, $Gr = \frac{g\beta(T_2 - T_0)h^3}{\nu^2}$ is the Grashof number, $We = \frac{U\Gamma}{h}$ is the Wesseinberg number, $N = \frac{2\alpha h}{\sqrt{k}}$ is the radiation parameter and $\text{Re} = \frac{Uh}{\nu}$ is the Reynolds number.

The corresponding dimensionless boundary conditions

$$u(0) = 0, \quad \theta(0) = r_T, \quad u(1) = 0, \quad \theta(1) = 1 \quad (8)$$

III. SOLUTION

Eq. (6) is non-linear and it is difficult to get a closed form solution. However for vanishing Wesseinberg number We, the boundary value problem is agreeable to an easy analytical solution. In this case the equation becomes linear and can be solved. Nevertheless, small Γ suggests the use of perturbation technique to solve the non-linear problem. Accordingly, we write

$$u = u_0 + We u_1 \quad (9)$$

and $\theta = \theta_0 + We\theta_1 \quad (10)$

Substituting equations (9) and (10) into Eqs. (6) and (7) and boundary conditions (8) and then equating the like powers of We, we obtain

A. *Zeroth-order system* (We^0)

$$\frac{d^2u_0}{dy^2} - M^2 u_0 = -\frac{Gr}{\text{Re}}\theta_0 \quad (11)$$

$$\frac{d^2\theta_0}{dy^2} + N^2\theta_0 = 0 \quad (12)$$

Together with boundary conditions

$$u_0(0) = u_0(1) = 0, \ \theta_0(0) = r_T, \ \theta_0(1) = 1 \quad (13)$$

B. *First-order system* (We)

$$\frac{d^2u_1}{dy^2} - M^2 u_1 = -\frac{d}{dy}\left[\left(\frac{du_0}{dy}\right)^2\right] - \frac{Gr}{\text{Re}}\theta_1 \quad (14)$$

$$\frac{d^2\theta_1}{dy^2} + N^2\theta_1 = 0 \quad (15)$$

Together with boundary conditions

$$u_1(0) = u_1(1) = 0, \ \theta_1(0) = 0, \ \theta_1(1) = 0 \quad (16)$$

C. *Zeroth-order solution*

Solving Eqs. (3.3) and (3.4) using the boundary conditions (3.8), we get

$$\theta_0 = A_1 \cos Ny + A_2 \sin Ny \quad (17)$$

$$u_0 = A_5 e^{-My} + A_6 e^{My} + \frac{Gr}{\text{Re}(N^2 + M^2)}(A_1 \cos Ny + A_2 \sin Ny) \quad (18)$$

here $A_1 = r_T$, $A_2 = \frac{1 - r_T \cos N}{\sin N}$,

$$A_3 = \frac{GrA_1}{\text{Re}(N^2 + M^2)},$$

$$A_4 = \frac{Gr}{\text{Re}(N^2 + M^2)}(A_1 \cos N + A_2 \sin N),$$

$$A_5 = \frac{A_4 - A_3 e^M}{e^M - e^{-M}}, \ A_6 = \frac{A_3 e^{-M} - A_4}{e^M - e^{-M}}.$$

D. *First-order solution*

Solving Eq. (3.7) subject to the boundary conditions in Eq. (3.8), we get

$$\theta_1 = 0 \quad (19)$$

Substituting the Eqs. (3.10) and (3.11) into the Eq. (3.6) and then solving the resulting equation with the corresponding conditions, we get

$$u_1 = \begin{pmatrix} c_1 e^{-My} + c_2 e^{My} + \frac{B_1}{3M^2} e^{-2My} - \frac{B_2}{3M^2} e^{2My} + \frac{B_3 e^{-My}\left(N^2 \cos Ny + 2MN \sin Ny\right)}{N^4 + 4N^2M^2} \\ + \frac{B_4 e^{-My}\left(N^2 \sin Ny - 2MN \cos Ny\right)}{N^4 + 4N^2M^2} - \frac{B_5 e^{My}\left(2MN \sin Ny - N^2 \cos Ny\right)}{N^4 + 4N^2M^2} \\ - \frac{B_6 e^{My}\left(N^2 \sin Ny + 2MN \cos Ny\right)}{N^4 + 4N^2M^2} - \frac{B_7 \sin 2Ny}{4N^2 + M^2} - \frac{B_8 \cos 2Ny}{4N^2 + M^2} \end{pmatrix} \tag{20}$$

Where, $c_1 = \dfrac{B_{10} - B_9 e^{M}}{e^{M} - e^{-M}}$,

$c_2 = \dfrac{B_9 e^{-M} - B_{10}}{e^{M} - e^{-M}}$, $B_1 = 2M^3 A_5^2$, $B_2 = 2M^3 A_6^2$,

$$B_3 = 2\frac{Gr A_5 MN^2\left(A_1 + A_2\right)}{\mathrm{Re}\left(N^2 + M^2\right)},$$

$$B_4 = 2\frac{Gr A_5 MN^2\left(A_2 - A_1\right)}{\mathrm{Re}\left(N^2 + M^2\right)},$$

$$B_5 = 2\frac{Gr A_6 MN^2\left(A_2 - A_1\right)}{\mathrm{Re}\left(N^2 + M^2\right)},$$

$$B_6 = 2\frac{Gr A_6 MN^2\left(A_1 + A_2\right)}{\mathrm{Re}\left(N^2 + M^2\right)},$$

$$B_7 = \frac{Gr^2 N^3\left(A_2^2 - A_1^2\right)}{\mathrm{Re}^2\left(N^2 + M^2\right)^2},$$

$$B_8 = 2\frac{Gr^2 N^3 A_1 A_2}{\mathrm{Re}^2\left(N^2 + M^2\right)^2},$$

$$B_9 = \begin{pmatrix} \frac{B_1}{3M^2} - \frac{B_2}{3M^2} + \frac{B_3 N^2}{N^4 + 4N^2M^2} - \frac{2B_4 MN}{N^4 + 4N^2M^2} \\ + \frac{B_5 N^2}{N^4 + 4N^2M^2} - \frac{2MNB_6}{N^4 + 4N^2M^2} - \frac{B_8}{4N^2 + M^2} \end{pmatrix}$$

and

$$B_{10} = \begin{pmatrix} \frac{B_1}{3M^2} e^{-2M} - \frac{B_2}{3M^2} e^{2M} + \frac{B_3 e^{-M}\left(N^2 \cos N + 2MN \sin N\right)}{N^4 + 4N^2M^2} \\ + \frac{B_4 e^{-M}\left(N^2 \sin N - 2MN \cos N\right)}{N^4 + 4N^2M^2} - \frac{B_5 e^{M}\left(2MN \sin N - N^2 \cos N\right)}{N^4 + 4N^2M^2} \\ - \frac{B_6 e^{M}\left(N^2 \sin N + 2MN \cos N\right)}{N^4 + 4N^2M^2} - \frac{B_7 \sin 2N}{4N^2 + M^2} - \frac{B_8 \cos 2N}{4N^2 + M^2} \end{pmatrix}.$$

Finally, the perturbation solutions up to first order for θ and u are given by

$$\theta = \theta_0 + We\theta_1 = A_1 \cos Ny + A_2 \sin Ny \tag{21}$$

and $u = u_0 + Weu_1$ (22)

The skin friction at the plate $y = 0$ of the channel is given by

$$\tau = \left.\frac{\partial u}{\partial y}\right|_{y=0} = \frac{\partial u_0}{\partial y} + We\frac{\partial u_1}{\partial y} \tag{23}$$

Where, $\dfrac{\partial u_0}{\partial y} = -MA_5 + MA_6 + \dfrac{GrNA_2}{\mathrm{Re}\left(N^2 + M^2\right)}$

and

$$\frac{\partial u_1}{\partial y} = \begin{pmatrix} -Mc_1 + Mc_2 - \frac{2B_1}{3M} - \frac{2B_2}{3M} + \frac{B_3\left(MN^2\right)}{N^4 + 4N^2M^2} + \frac{B_4\left(2M^2N\right)}{N^4 + 4N^2M^2} \\ + \frac{B_4\left(N^3\right)}{N^4 + 4N^2M^2} - \frac{B_5\left(MN^2\right)}{N^4 + 4N^2M^2} - \frac{B_6\left(2M^2N\right)}{N^4 + 4N^2M^2} \\ - \frac{B_6\left(N^3\right)}{N^4 + 4N^2M^2} - \frac{2NB_7}{4N^2 + M^2} \end{pmatrix}$$

The rate of heat transfer coefficient in terms of Nusselt number Nu at the plate $y = 0$ of the channel is given by

$$Nu = \left.\frac{\partial \theta}{\partial y}\right|_{y=0} = NA_2 \tag{24}$$

IV. RESULTS AND DISCUSSIONS

In order to perceive the effects of various physical parameters like Gr, Re, We, M, N and r_T on the velocity profile u we have plotted the Figs. 2-7. From Fig. 2, it is observed that the velocity u first decreases and then increases with increasing Wesseinberg number We. From Fig. 3, it is seen that the velocity u decreases with increasing Hartmann number M. From Fig. 4, it is found that the velocity u increases with an increase in radiation parameter N. From Fig. 5, it is observed that the velocity u increases with increasing Grashof number Gr. From Fig. 6, it is noted that, the velocity u decreases with an increase in Reynolds number Re. From Fig. 7, it is found that, the velocity u increases with increasing r_T.

Table-1 depicts the effects of various physical parameters like Gr, Re, We, M, N and r_T on the skin friction τ at the plate $y = 0$. It is found that, the skin friction τ increases with increasing radiation parameter N, Grashof number Gr and r_T, while it decreases with increasing Wesseinberg number We, Hartmann number M and Reynolds number Re.

Fig. 8 shows the effect of wall temperature parameter r_T on θ. It is observed that, the temperature θ increases with an increase in r_T.

Fig. 9 depicts the effect of radiation parameter N on θ. It is observed that, the temperature θ increases with an increase in r_T.

Table-2 shows the effect of r_T on the Nusselt number Nu at the plate $y = 0$. It is found that, the Nusselt number increases with increasing radiation parameter N, while it decreases with increasing r_T.

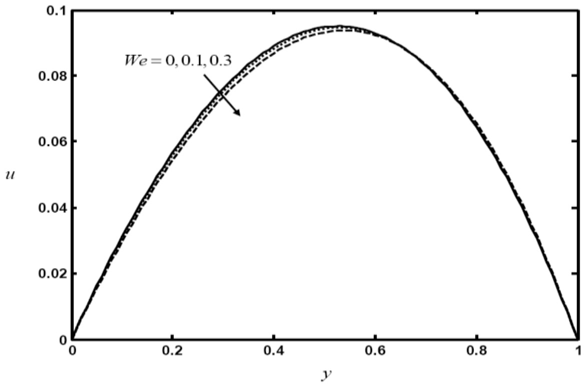

Fig. 2. Effect of Weissenberg number We on u for $Gr = 1$, $N = 1$, $M = 1$, $r_T = 0.5$ and $\mathrm{Re} = 1$.

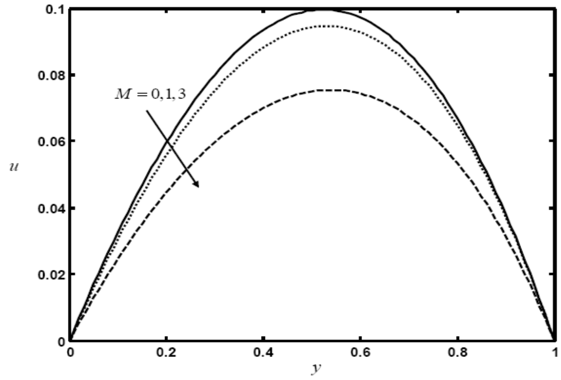

Fig. 3. Effect of Hartmann number M on u for $Gr = 1$, $N = 1$, $We = 0.1$, $r_T = 0.5$ and $\mathrm{Re} = 1$.

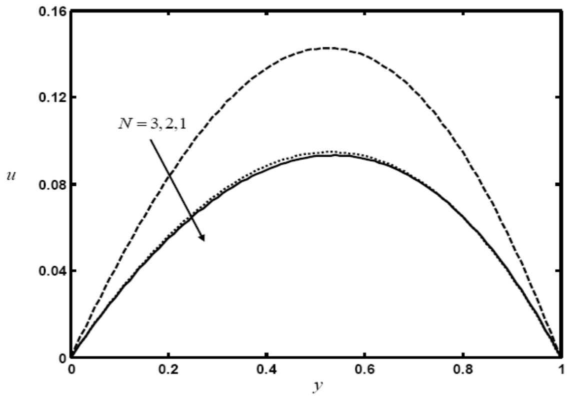

Fig. 4. Effect of radiation parameter N on u for $Gr = 1$, $We = 0.1$, $M = 1$, $r_T = 0.5$ and $\mathrm{Re} = 1$.

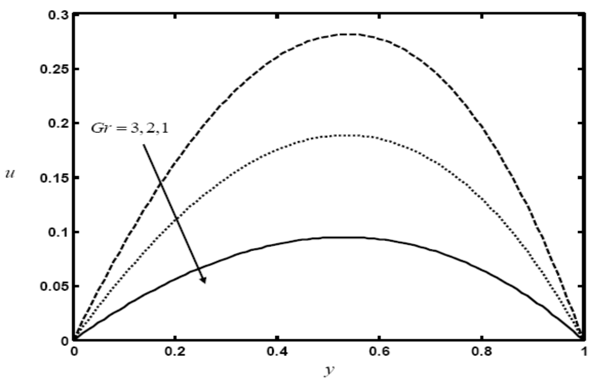

Fig. 5. Effect of Grashof number Gr on u for $N = 1$, $We = 0.1$, $M = 1$, $r_T = 0.5$ and $\mathrm{Re} = 1$.

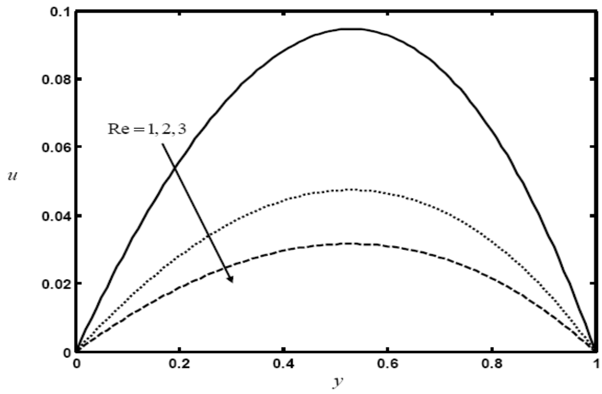

Fig. 6. Effect of Reynolds number Re on u for $Gr = 1$, $We = 0.1$, $M = 1$, $r_T = 0.5$ and $N = 1$.

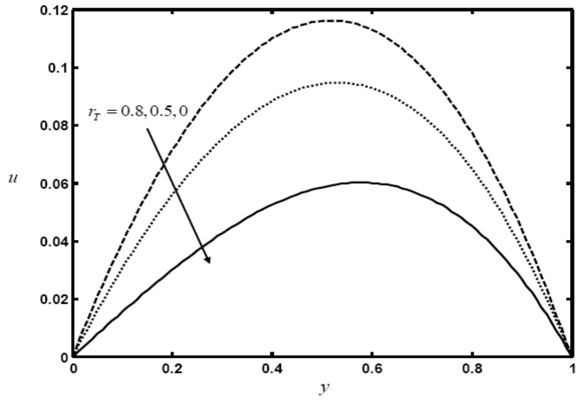

Fig. 7. Effect of r_T on u for $Gr = 1$, $We = 0.1$, $M = 1$, $N = 1$ and $\mathrm{Re} = 1$.

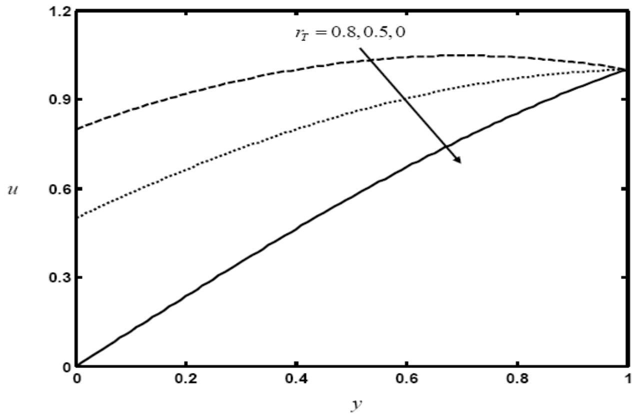

Fig. 8. Effect of wall temperature parameter r_T on θ for $N = 1$.

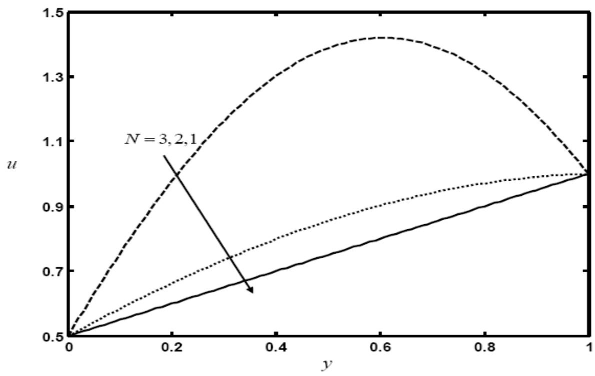

Fig. 9. Effect of radiation parameter N on θ for $r_T = 0.5$.

Table – 1: The Skin friction τ at the plate $y=0$

We	M	N	Gr	Re	r_T	τ
0.1	1	1	1	1	0.5	0.3305
0.2	1	1	1	1	0.5	0.3245
0.1	2	1	1	1	0.5	0.2705
0.1	1	2	1	1	0.5	0.4745
0.1	1	1	2	1	0.5	0.6491
0.1	1	1	1	2	0.5	0.1667
0.1	1	1	1	1	0.8	0.4331

Table – 2: The Nusselt number Nu at the plate $y=0$

N	r_T	Nu
1	0.5	0.8673
2	0.5	2.6572
1	0.8	0.6747

V. CONCLUSION

In this chapter, we studied the MHD fully developed free convection flow of a Williamson fluid in a vertical channel with radiative heat transfer. The governing non-linear equations are solved for the velocity field and temperature field using the perturbation technique. It is found that, the velocity increases with increasing N, Gr and r_T, while it decreases with increasing We, M and Re; the skin friction τ increases with increasing α, Gr and r_T, while it decreases with increasing We, M and Re; the temperature θ increases with an increase in r_T and the Nusselt number decreases with increasing r_T.

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Mrs.B.Swaroopa, M.Sc.,M.Phil.,NET ,(Ph.D)

Publications :

- Effect of heat transfer on oscillatory flow of a fluid through a porous medium in a channel with an inclined magnetic field
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Research work : Persuing Ph.D and submitted the Synopsis
Membership : As a member in GCTA – A.P.
Achievements :

- Secured 2 nd rank both in S.V.U. and S.K.U CET
- Stood University Second in M.Sc
- Stood College First in B.Sc
- One of my students secured State Level Heighest marks in B.Sc