International Journal of Engineering and Technical Research (IJETR)
ISSN: 2321-0869 (O) 2454-4698 (P), Volume-3, Issue-10, October 2015

Radiation effects on heat and mass transfer in a steady MHD flow over a porous vertical plate

Vibhor Tomer, Manoj Kumar

***Abstract*— A steady two dimensional mixed convective magnetic hydrodynamic flow over a porous vertical plate has been considered. The governing equations and boundary conditions are non dimensionalized then transformed by one point group transformation and finally solved numerically by using Runge-Kutta fourth-fifth order numerical method with shooting technique. Effects of radiation parameter, suction/ injection, porosity, magnetic parameter, Schmidt number and Prandtl number on velocity, temperature and concentration profile are discussed and presented graphically. It is observed that velocity and temperature decrease while concentration increases with an increase in radiation parameter. The values of physical quantities skin friction coefficient, Nusselt number and Sherwood number are also tabulated with the variation of physical parameters.**

***Index Terms*— Heat transfer, MHD, Mass transfer, porosity, Radiation**

I. INTRODUCTION

The effects of radiation on a MHD fluid flow due to a stretching/ shrinking surface have great interest of researchers. **Chamkha [2]** investigated the coupled heat and mass transfer by natural convection of Newtonian fluids in the presence of magnetic field and radiation effects. **Mahmoud [9]** presented a study of the flow and heat transfer of an incompressible viscous electrically conducting fluid over a continuously moving vertical infinite plate with uniform suction and heat flux in the presence of radiation taking into account the effects of variable viscosity. **Cortell [3]** presented a numerical analysis for flow and heat transfer in a viscous fluid over a sheet nonlinearly stretched with effects of thermal radiation. Radiation effect on fluid flow with and without applying a magnetic field have been investigated among others: **Bataller [1], Hsiao [6], Jat and Chaudhary [7]** etc. **Mahanti and Gaur [8]** investigated the effects of viscosity and thermal conductivity which vary linearly on steady free convective flow of a viscous incompressible fluid along an isothermal vertical plate in the presence of heat sink. **Seddeek et al. [10]** have presented similarity representation of MHD flow with heat transfer taking into consideration variable viscosity and thermal conductivity.

Vibhor Tomer, Department of Mathematics, Statistics and Computer Science, G. B. Pant University of Agriculture & Technology, Pantnagar – 263145, Uttarakhand, India, 9457031801

Manoj Kumar, Department of Mathematics, Statistics and Computer Science, G. B. Pant University of Agriculture & Technology, Pantnagar – 263145, Uttarakhand, India, 7500241423

Hamad *et al.* [5] studied a steady laminar 2-D MHD viscous incompressible flow over a permeable flat plate with thermal convective boundary condition, radiation effects and similarity representation of the governing partial differential equations obtained by group method. **Ferdows *et al.* [4]** investigated a free convective heat and mass transfer flow over a moving permeable flat vertical stretching sheet in the presence of non-uniform magnetic field. Effects of thermal radiation and convective surface boundary condition on steady boundary layer flow of a viscous incompressible electrically conducting fluid are considered. A scaling group of transformation is applied to the governing equations and the boundary conditions.

The objective of present investigation is to study radiation effects on MHD flow over a porous vertical plate. To finding the solution, authors are using scaling group method of transformation. The attempt has also been made to study the effects of radiation, suction/ injection, porosity, thermal radiation, magnetic parameter, Schmidt number, Prandtl number on the fluid flow and the rate of heat and mass transfer.

II. MATHEMATICAL FORMULATION OF THE PROBLEM

Consider a two dimensional steady laminar flow of an incompressible MHD fluid over a vertical porous plate in presence of constant suction and heat source and transverse magnetic field. Let the $\bar{x}$-axis be taken in vertically upward direction along the plate and $\bar{y}$-axis normal to it. The physical sketch and geometry of the problem is shown in Figure 1. The fluid velocity $\bar{u}$ and $\bar{v}$ are along $\bar{x}$ and $\bar{y}$-axis respectively. A magnetic field B is applied in the $\bar{y}$-direction that is normal to the flow direction. Suction or injection is imposed on the porous plate. The temperature of the surface is held uniform at T_w which is higher than the ambient temperature T_∞. The species concentration at the surface is maintained uniform at $C_w = 1$ while the ambient fluid concentration is assumed to be C_∞. The governing fluid flow equations of continuity, momentum, heat transfer and mass transfer respectively are as follows:

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = 0 \tag{1}$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = \left(\nu + \frac{\kappa}{\rho}\right)\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \frac{\sigma B^2}{\rho}\bar{u} - \frac{\mu \bar{u}}{\rho K} \tag{2}$$
$$+ g\beta_T (T - T_\infty) + g\beta_C (C - C_\infty)$$

$$\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = \frac{k^*}{\rho C_P}\frac{\partial^2 T}{\partial \bar{y}^2} - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial \bar{y}} + \frac{\mu}{\rho C_p}\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)^2 \tag{3}$$

$$\bar{u}\frac{\partial C}{\partial \bar{x}} + \bar{v}\frac{\partial C}{\partial \bar{y}} = D\frac{\partial^2 \bar{C}}{\partial \bar{y}^2} \tag{4}$$

The boundary conditions are given by

$$\bar{u} = u_w = \bar{x}, \bar{v} = -v_w, C = C_w, T = T_w \quad \text{at } \bar{y} = 0 \qquad (5a)$$

$$\bar{u} \to 0, T \to T_\infty, C \to C_\infty \qquad \text{as } \bar{y} \to \infty \qquad (5b)$$

Here $\bar{u}$ and $\bar{v}$ are the velocity components in $\bar{x}$ and $\bar{y}$ directions, respectively, ν is the kinematic coefficient of viscosity, μ is coefficient of viscosity, κ is vortex viscosity, σ is electrical conductivity of the fluid, K is porosity parameter, ρ is fluid density, T is fluid temperature, β_T is thermal expansion coefficient, β_C is concentration expansion coefficient, k^* is the thermal conductivity, C_p is specific heat, D is mass diffusivity.

By using Rosseland's approximation, the radiative heat flux q_r is given by

$$q_r = -\frac{4\sigma_1}{3k_1}\frac{\partial T^4}{\partial \bar{y}} \qquad (6)$$

Here σ_1 is the Stefan-Boltzman constant and k_1 is the absorption coefficient. It is assumed that the temperature variation within the flow is such that T^4 may be expanded in a Taylor series about T_∞ and neglecting higher order terms, we get

$$T^4 \cong 4TT_\infty^3 - 3T_\infty^4 \qquad (7)$$

Equations (6) and (7) give

$$\frac{\partial q_r}{\partial \bar{y}} = -\frac{16\sigma_1 T_\infty^3}{3k_1}\frac{\partial^2 T}{\partial \bar{y}^2} \qquad (8)$$

Using equation (8) the energy equation (3) becomes

$$\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = \frac{k^*}{\rho C_P}\frac{\partial^2 T}{\partial \bar{y}^2} - \frac{16\sigma_1 T_\infty^3}{3\kappa_1 C_p \rho}\frac{\partial^2 T}{\partial \bar{y}^2} + \frac{\mu}{\rho C_p}\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)^2 \qquad (9)$$

A. Nondimensionalization

Introducing the following dimensionless variables as considered by Hamad et al. (2012):

$$x = \frac{\bar{x}}{L},\ y = \frac{\bar{y}\sqrt{Re}}{L},\ u = \frac{\bar{u}}{u_\infty},\ \bar{v} = \frac{v\sqrt{Re}}{u_\infty}, \qquad (10a)$$

$$\theta = \frac{T - T_\infty}{T_f - T_\infty},\ \varphi = \frac{C - C_\infty}{C_w - C_\infty}$$

$$u = \frac{\partial \psi}{\partial y},\ v = -\frac{\partial \psi}{\partial x} \qquad (10b)$$

where $Re = u_\infty L/\nu$ is the Reynolds number, ψ is the stream function, L being the characteristic length and u_∞ is reference velocity.

Hence, equations (2), (9) and (4) reduce in the following form:

$$\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial y^2} - (1+K)\frac{\partial^3 \psi}{\partial y^3} + (M + K_p)\frac{\partial \psi}{\partial y} \qquad (11)$$
$$-\frac{1}{Re^2}(Gr\theta + \varphi Gr) = 0$$

$$\frac{\partial \psi}{\partial y}\frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial \theta}{\partial y} - \frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2} - \frac{4}{3}\frac{1}{RPr}\frac{\partial^2 \theta}{\partial y^2} - Ec\left(\frac{\partial^2 \psi}{\partial y^2}\right)^2 = 0 \qquad (12)$$

$$\frac{\partial \psi}{\partial y}\frac{\partial \varphi}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial \varphi}{\partial y} - \frac{1}{Sc}\frac{\partial^2 \varphi}{\partial y^2} = 0 \qquad (13)$$

Subject to boundary conditions

$$\frac{\partial \psi}{\partial y} = x,\ \frac{\partial \psi}{\partial x} = -\frac{Vw}{u_\infty \sqrt{Re}},\ \theta = 1,\ \varphi = 1 \text{at } y = 0 \qquad (14a)$$

$$\frac{\partial \psi}{\partial y} \to 0,\ \theta \to 0,\ \varphi \to 0 \qquad \text{as } y \to \infty \qquad (14b)$$

Here $K = \frac{\kappa}{\mu}$ is material parameter, $M = \frac{\sigma B^2 L}{\rho u_\infty}$ is the Hartmann number, $K_P = \frac{L\mu}{U_\infty \rho K'}$ is the porosity parameter, $Gr = \frac{gL^3\beta_T(T_w - T_\infty)}{\upsilon^2}$ and $Gc = \frac{gL^3\beta_C(C_w - C_\infty)}{\upsilon^2}$ are thermal and mass Grashof number respectively, $R = \frac{k_1 k^*}{4T_\infty^3\sigma}$ is the conduction-radiation parameter, $Pr = \frac{\mu C_p}{k^*}$ is the Prandtl number, $Ec = \frac{U_\infty^2}{C_p(T_w - T_\infty)}$ is the Eckert number, $Sc = \frac{\nu}{D}$ is the Schmidt number.

B. Group Transformation

The application of group transformations has been considered to find similarity reduction of equations (11), (12) and (13). Consider the following group transformations

$$\Gamma : x^\# = x\Omega^{\alpha_1},\ y^\# = y\Omega^{\alpha_2},\ \psi^\# = \psi\Omega^{\alpha_3}, \qquad (15)$$
$$\theta^\# = \theta\Omega^{\alpha_4},\ \varphi^\# = \varphi\Omega^{\alpha_5}$$

where α_1, α_2, α_3, α_4 and α_5 are constants and Ω is the parameter of point transformation. Now finding the relation among α's such that

$$\Delta_j(x^\#, y^\#, \psi^\#, \theta^\#, \varphi^\#, \ldots, \frac{\partial^3 \psi^\#}{\partial y^{\#3}}) = H_j(x, y, \psi, \theta, \varphi, \ldots, \frac{\partial^3 \psi}{\partial y^3})$$

$$\varphi_j(x, y, \psi, \theta, \varphi, \ldots, \frac{\partial^3 \psi}{\partial y^3})(j = 1,2,3)$$

Δ_1, Δ_2 and Δ_3 are conformally invariant under the group transformation (15).

By using above group transformation in equation (11) and solving the resulting equations one finds the following

$$\alpha_2 = \alpha_4 = \alpha_5 = 0 \text{ and } \alpha_1 = \alpha_3 \qquad (16)$$

Similarly equations (12), (13) and (14) are also giving $\alpha_1 = \alpha_3$, $\alpha_2 = \alpha_4 = \alpha_5 = 0$, so these equations show invariant under the group transformation (15).

Now the characteristic equations are

$$\frac{dx}{x} = \frac{dy}{0} = \frac{d\psi}{\psi} = \frac{d\theta}{0} = \frac{d\varphi}{0} \qquad (17)$$

which give the following similarity transformations:

$$\eta = y,\ \psi = xf(\eta),\ \theta = \theta(\eta),\ \text{and } \varphi = \varphi(\eta) \qquad (18)$$

Using these transformations, the momentum, energy and mass equations become

$$f''' = \frac{1}{1+K}\left[\begin{array}{l}(f'^2 - ff'') + (M + K_P)f' - \\ \frac{\cos\alpha}{Re^2}(Gr\theta + \varphi Gc)\end{array}\right] \qquad (19)$$

$$\theta'' = \frac{-3RPr}{3R+4}\left(f\theta' + Ecf''^2\right) \tag{20}$$

$$\varphi'' = Sc(\varphi c - f\varphi') \tag{21}$$

Subject to the boundary conditions

$$f = f_w,\ f' = 1,\ \theta = 1,\ \varphi = 1 \quad \text{at} \quad \eta = 0$$

$$f' \to 0,\ \theta \to 0,\ \varphi \to 0 \quad \text{as} \quad \eta \to \infty \tag{22}$$

The physical quantities of interest are the Skin friction coefficient C_f, Nusselt number Nu and Sherwood number Sh, which are defined as

$$C_f = \frac{\mu}{\rho \bar{u}^2}\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)_{\bar{y}=0}, \quad Nu = \frac{-\bar{x}}{T_w - T_\infty}\left(\frac{\partial T}{\partial \bar{y}}\right)_{\bar{y}=0},$$

$$Sh = \frac{-\bar{x}}{C_w - C_\infty}\left(\frac{\partial C}{\partial \bar{y}}\right)_{\bar{y}=0}$$

III. Method of solution

The system of ordinary differential equations (19), (20) and (21) subject to the boundary conditions (22) have been solved numerically using Runge-Kutta method with shooting technique. The computations we are carried out using step size of $\Delta\eta = 0.01$ selected to be satisfactory for a convergence criterion of 10^{-6} in all cases.

The physical quantities skin friction coefficient C_f, Nusselt number Nu and Sherwood number Sh indicate the wall shear stress, rate of heat transfer and rate of mass transfer respectively and these are proportional to the numerical values of $f''(0)$, $-\theta'(0)$ and $-\varphi'(0)$ respectively.

IV. Results and discussion

The numerical results for velocity, temperature, concentration, wall heat transfer, the rate of heat and mass transfer have been computed and represented. The computations are carried out for different values of different parameters for suction f_w(= 0.5, 1), for injection f_w(= -0.1, -0.5, -1). The calculations were made by taking radiation parameter ($0.2 \le R \le 1$), Sc = (0.22, 0.72, 6.8, 10) with Gr = 0.1, Gc = 0.1, Ec = 1, Re = 1 and Prandtl number Pr = 0.72 for air at 1 atmospheric pressure. Different values of magnetic parameter, i.e. $0.1 \le M \le 1$ were taken to analyse the study. The values for the skin friction coefficient, Nusselt number and Sherwood number have been tabulated in table 1. It is seen that the value of skin friction coefficient varies and is greater for suction while smaller for injection. Nusselt number is less for suction in comparison to injection and Sherwood number is higher for suction while smaller for injection.

Table 1: Numerical values of $f''(0)$, $\theta'(0)$, $\varphi'(0)$ for different values of non dimensional parameters $f_w = 0.5$, K = 0.1, Sc = 0.5, Re = 1, Gr = 0.1, Gc = 0.1 and Ec = 1.

R	K_p	M	Pr	$f''(0)$	$\theta'(0)$	$\varphi'(0)$
0.2	0.1	0.5	0.72	0.097521	-0.149050	-0.522674
0.5	0.1	0.5	0.72	0.047521	-0.231590	-0.515161
1	0.1	0.5	0.72	0.000014	-0.295051	-0.507571
1	0.4	0.5	0.72	0.352910	-0.286591	-0.530115
1	0.7	0.5	0.72	0.838471	-0.241011	-0.563746
1	0.1	0.1	0.72	-0.452104	-0.218399	-0.464637
1	0.1	1	0.72	0.659101	-0.334052	-0.551361
1	0.1	0.5	0.22	0.075901	-0.143201	-0.519201
1	0.1	0.5	6.8	0.010013	-1.327659	-0.511292
1	0.1	0.5	10	0.001004	-1.746591	-0.510820
1	0.1	0.5	0.72	0.000014	-0.286051	-0.504839
1	0.1	0.5	0.72	0.001020	-0.309251	-0.511870
1	0.1	0.5	0.72	0.001001	-0.298502	-0.689439
1	0.1	0.5	0.72	0.015202	-0.306302	-0.863251

Fig. 2–4 exhibit the effects of suction on velocity, temperature and concentration. It is seen that the velocity, temperature and concentration decreases with an increase in suction parameter. Fig. 5–6 show the effect of injection on velocity and concentration profile. Velocity first increases then sharply decreases while concentration increases with rising as the value of injection. Fig.7-8 depict the effect of radiation parameter on velocity and temperature profile, it has been seen that velocity and temperature decreases with its increasing values (for R = 0.2, 0.5, 1). It is observed that velocity and temperature decrease while concentration increases with an increase in radiation parameter. Fig.9 shows the effect of magnetic parameter on velocity. Velocity increases as the value of M increases. The effect of Prandtl number Pr on temperature is represented by fig.10. It is observed that the velocity sharply decreases with its increasing values (for Pr = 0.22, 6.8, 10). The effect of Schmidt number Sc on concentration is represented through fig. 11. It has been observed that concentration decreases as Schmidt number increases.

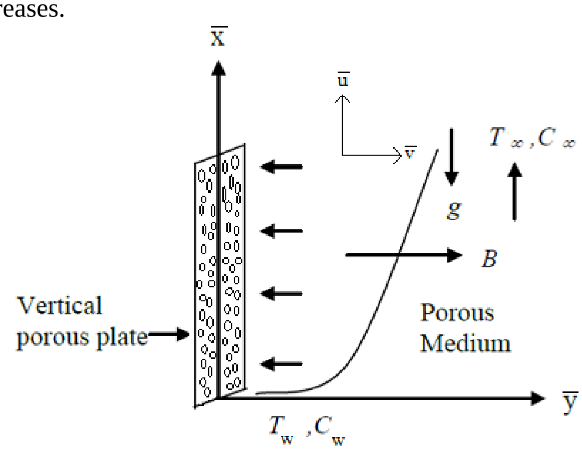

Fig.1: Physical representation of fluid flow

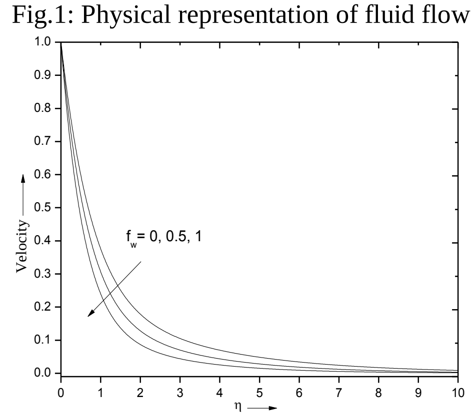

Fig.2: Effect of suction on velocity

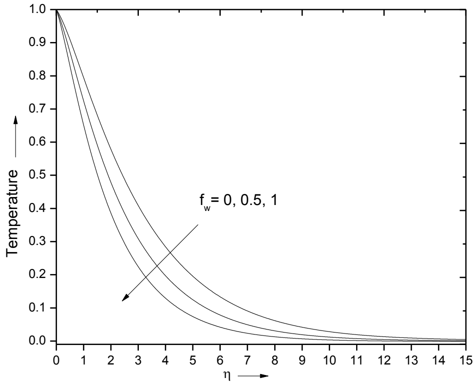

Fig.3: Effect of suction on temperature

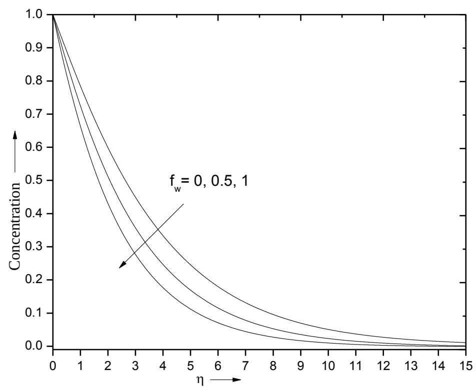

Fig.4: Effect of suction on concentration

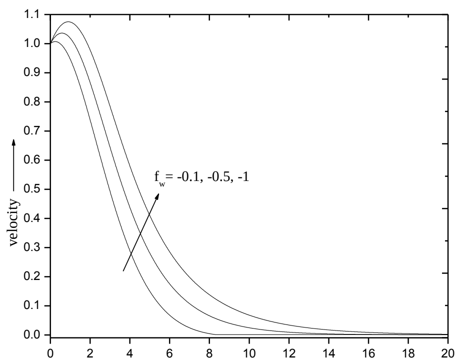

Fig.5: Effect of injection on velocity

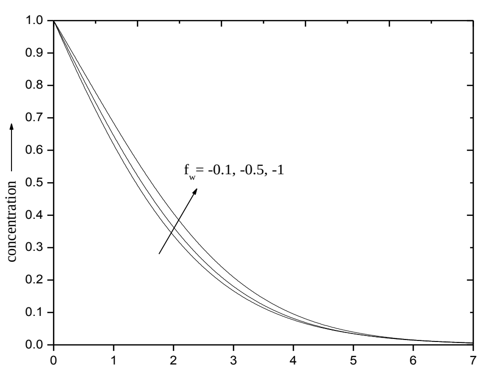

Fig.6: Effect of injection on concentration

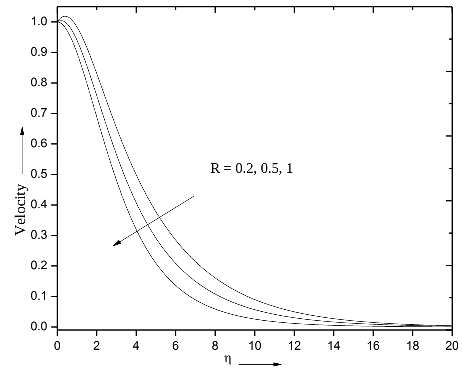

Fig.7: Effect of radiation on velocity

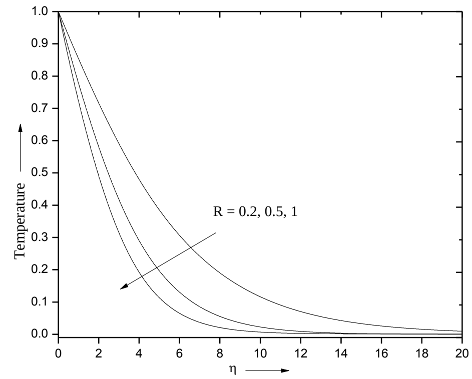

Fig.8: Effect of suction on temperature

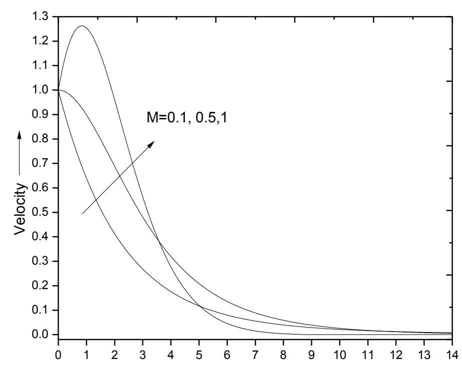

Fig.9: Effect of magnetic parameter on velocity

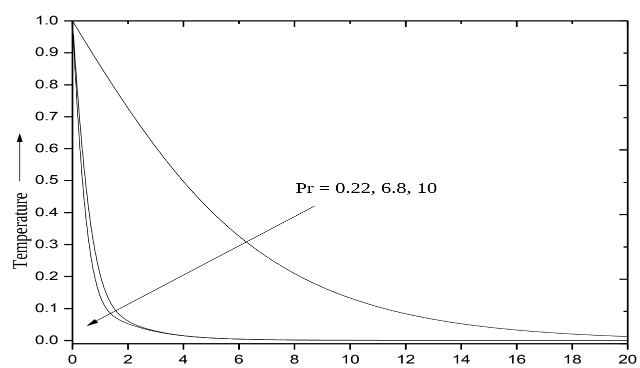

Fig.10: Effect of Prandtl number on temperature

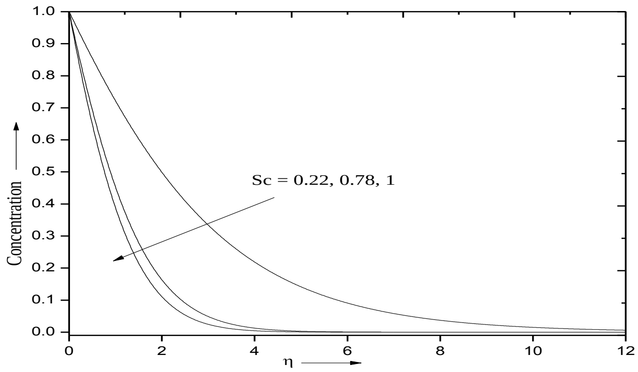

Fig.11: Effect of Schmidt number on concentration

REFERENCES:

[1] Bataller R.C, Radiation effects on Blasius flow. Appl. Math. Comput. 198(2008), 333–338.

[2] Chamkha A.J, Coupled heat and mass transfer by natural convection about a truncated cone in the presence of magnetic field and radiation effects. Numer. Heat Transfer A: Appl. 39(2001), 511–530.

[3] Cortell R, Effects of viscous dissipation and radiation on thermal boundary layer over a nonlinearly stretching sheet. Phys. Lett. A 372(2008), 631–636.

[4] Ferdows M, Uddin M.J, Afify A.A, Scaling group transformation for MHD boundary layer free convective heat and mass transfer flow past a convectively heated nonlinear radiating stretching sheet. International Journal of Heat and Mass Transfer 56(2013), 181–187

[5] Hamad M.A.A, Uddin J.U, Ismail A.I.M, Radiation effects on heat and mass transfer in MHD stagnation-point flow over a permeable flat plate with thermal convective surface boundary condition, temperature dependent viscosity and thermal conductivity. Nuclear Engineering and Design. 242(2012), 194–200.

[6] Hsiao K.L, Mixed convection with radiation effect over a nonlinearly stretching sheet. World Acad. Sci. Eng. Technol. 62(2010), 242–338.

[7] Jat R.N, Chaudhary S, Radiation effects on the MHD flow near the stagnation point of a stretching sheet. ZAMP. 33(2010), 2–5.

[8] Mahanti N.C, Gaur P, Effects of varying viscosity and thermal conductivity on steady-free convective flow and heat transfer along an isothermalvertical plate in the presence of heat sink. J. Appl. Fluid Mech. 2(2009), 23–28.

[9] Mahmoud M.A.A, Variable viscosity effects on hydromagnetic boundary layer flow along a continuously moving vertical plate in the presence of radiation. Appl. Math. Sci. 1(2007), 799–814.

[10] Seddeek M.A, Afify A.A, Hanaya M.A, Similarity solutions for steady MHD Falkner–Skan flow and heat transfer over a wedge by considering the effect of variable viscosity and thermal conductivity. Applic. Appl. Math. 4(2009), 301-313.

Authors' information

**Dr. Vibhor Tomer**

- Assistant Professor at the Department of Mathematics, UTU Dehradun, India
- Ph.D. from G.B.P.U.A. & T. Pantnagar in 2014
- Qualified CSIR-NET in Mathematical Sciences in June 2012
- M.Phil. in Mathematics in 2008
- M.Sc. in Mathematics from C.C.S. University, Meerut in 2005.

**Dr. Manoj Kumar**

- Professor at the Dept. of Mathematics Statics and Computer Science, G.B. Pant University of Agriculture and technology, Pantnagar, India
- Ph.D. From Indian Institute of Technology, Roorkee
- **Research Activities:** More than 30 Post Graduate students guided successfully including 8 Ph. D.
- **Publications:** More than 30 Research papers in referred International Journals.