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New Modification for Slope – Deflection Equation in Structural Analysis

Muhammed Abbas Husain

***Abstract*— The slope – deflection method is one of the classical displacement methods which is used to analyze the statically indeterminate beams and frames. This method is based on applying special equations and equilibrium equations, and then solving a system of simultaneous equations to obtain the results. A modified equation for the slope – deflection method is used in some special cases of end spans to reduce the computations. In this study, a new modified equation for the slope – deflection method is presented. The new modified equation replaces the "old" modified equation by a more applicable equation, and a more superior equation which can includes more cases and also reduce the long computations to analyze the structures. A numerical example has been solved by using the basic slope – deflection equation and then solved again by using the presented new modified equation. A comparison between the two procedures has been achieved. It has been concluded that for the cases of pinned or roller end spans having a couple moment at their ends, and for overhanging spans, using of "New modified slope – deflection equation" reduces the long computations and makes the analysis more simpler than the basic slope – deflection equation.**

***Index Terms*— statically indeterminate, slope – deflection, modified equation, new modification.**

I. Introduction

The analysis of statically indeterminate structures can be classified generally into two categories: namely, the force (or flexibility) methods, and the displacement (or stiffness) methods. The slope – deflection is one of the displacement methods. The slope – deflection method was first introduced by George A. Maney in 1915. This method is used to analyze the statically indeterminate beams and frames, and it based on special equation which relates the member end moments to the displacements and rotations at the ends of the members.
For each member, a basic slope – deflection equations are applied at each end (near and far ends), also an equilibrium equations for specified structural joints are applied , then the analysis of the structure is achieved by solving the system of simultaneous equations. For some cases (when the end span of the beam or frame is supported by a pin or roller at its far end), to reduce the computations, one slope – deflection is adequate for the span applied at fixed end. In these cases this one slope – deflection equation is called the "modified slope – deflection equation". The main advantages for using the slope – deflection method are the ease of programing and the wide range applicability for the indeterminate structures.

The slope – deflection method is still used in quick analysis, preliminary design, checking the analysis, and for the analysis of small structures. Also the textbooks and the references in "Structural Analysis" are still give an importance area for this method.

Muhammed Abbas Husain, Department of civil and environmental engineering / College of engineering / University of Samarra, Samarra, Iraq.

II. Slope – Deflection Equation

A. Basic Slope – Deflection Equation:

For a typical member AB shown in Fig.1:

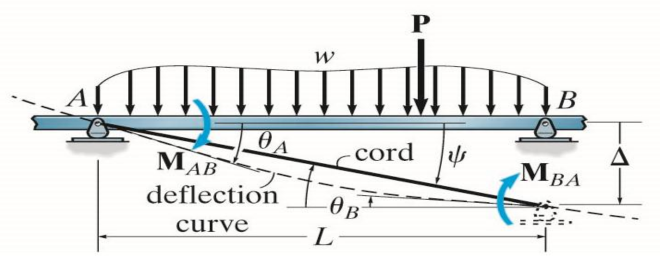

Fig.1: Typical beam member. Ref.[1]

If the ends A and B are considered to be as "near end", and "far end", and denoted by the letters "N" and "F" respectively, then the basic slope – deflection equation is:

$$M_N = 2Ek\left[2\theta_N + \theta_F - 3\psi\right] + FEM_N$$

where

M_N = internal moment in the near end.
E = modulus of elasticity of material.
k = member stiffness , equals I / L.
I = moment of inertia for the section about the neutral axis.
L = span length.
θ_N = the slope at the near end.
θ_F = the slope at the far end.
ψ = member rotation , equals Δ / L.
Δ = relative vertical displacement between the two ends.
FEM_N = fixed end moment at the near end.

This equation is generally applied twice for each member, the first application is for the near end and the second application is for the far end.
For the far end, the basic slope – deflection becomes:

$$M_F = 2Ek\left[2\theta_F + \theta_N - 3\psi\right] + FEM_F$$

This equation is similar to the first equation but the difference is that the subscripts N and F are interchanged.

To analyze the statically indeterminate beams or frames, the basic slope – deflection equation is applied twice for each member (at near and far ends), then an equilibrium equations are applied for specified joints to complete a system of simultaneous equations, which then solved to obtain the member end moments, which lead to full analysis of the structure.

B. Modified Slope – Deflection Equation:

When the end span of a beam or frame is supported at its far end by a pin or roller with no external moment is applied, such as the support B in Fig.2 shown below, the basic slope – deflection equation can be reduced to a modified slope – deflection equation.

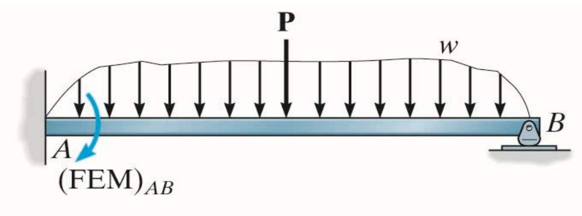

Fig.2: Typical pin end span. Ref.[1]

The modified slope – deflection equation is:

$$M_N = 3Ek\left[\theta_N - \psi\right] + FEM_N$$

This equation is called the "modified slope – deflection equation".

This equation is obtained by applying the basic slope – deflection equation twice, as explained before, but with substituting the value of M_F with zero since the pin or roller end does not exert a moment, then the two simultaneous equations are solved to give the modified equation.

Using of this modified equation reduces the computations and eliminates one equation for each end span with pinned or roller ends provided no external moment at these ends.

III. New Modified Slope – Deflection Equation

The modified slope – deflection equation gives a simpler solution for the cases of end spans with pin or roller supports at their ends "provided no external moments at that ends". If there is an external moment at the pinned or roller ends, or even for supports with an overhang cantilever ends, the modified slope – deflection equation becomes not applicable and it is required to come back to the basic slope – deflection equation and apply it twice to solve the problem.

In this paper a "new" slope – deflection equation is presented to simplify the solution for the end members with pin or roller supports at their ends "even with external moments at these ends".

The "new" modified slope – deflection equation is similar to the "old" modified slope – deflection equation, but the only difference between them is that the carry over moment (COM) is added to the right side of the "old" modified equation. This addition of COM represents the additional moment that added (or carried over) to the near end from the far end (when the external moment is applied).

The carry over moment (COM) is the moment that developed at one end of the member due to the external moment (M) that applied at the other end.

There are two values of COM as follows,

$$COM = \begin{cases} M/2 & \text{if far end of the member is fixed} \\ 0 & \text{if far end of the member is pinned} \end{cases}$$

The new modified slope – deflection equation is:

$$M_N = 3Ek\left[\theta_N - \psi\right] + FEM_N + COM$$

This new slope – deflection equation will simplify and reduce the computations for wide - range types of structures compared with the "old" slope – deflection equation. The end members that have pinned or roller supports at their ends with or without external moments at these ends, also the members with pinned or roller supports with an overhang cantilever ends, all can be easily solved by this new equation.

It can be noticed that if no external moment applied at the member end, the new and old equations will be identical.

IV. Numerical Example

The following example illustrates the reduction in solution by using the new modified slope – deflection equation.

This example is presented in Ref. [2], it is a continuous statically indeterminate beam with an overhang cantilever end as shown in Fig. 3(a).

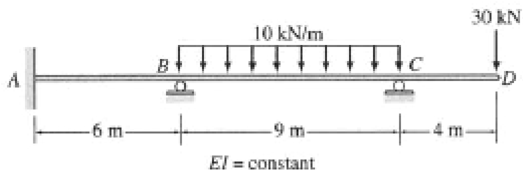

Fig. 3(a) : Continuous beam. Ref. [2]

The cantilever end *CD* can be replaced by a shear force and a moment at end *C* as shown in the Fig. 3(b) and Fig. 3(c).

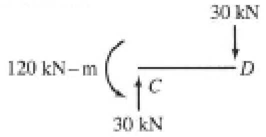

Fig. 3(b) : Cantilever portion. Ref. [2]

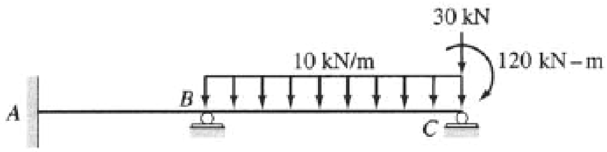

Fig. 3(c) : Equivalent beam. Ref. [2]

This example will be solved in two approaches. The first approach represents the solution by using the basic slope – deflection equation (this solution is presented in Ref. [2]). The second approach represents the solution by using the new modified slope – deflection equation presented in this paper.

It can be noticed that the old modified slope – deflection equation does not applicable for the end span *BC* since there is an external moment at the roller C.

A. Solution I (By using the basic slope – deflection equation – presented in Ref.[2]) :

The solution steps are as follows :

Step 1: Finding the fixed end moments :

$$FEM_{AB} = 0$$
$$FEM_{BA} = 0$$
$$FEM_{BC} = -\frac{wl^2}{12} = -\frac{10(9)^2}{12} = -67.5\ kN.m$$
$$FEM_{CB} = \frac{wl^2}{12} = \frac{10(9)^2}{12} = 67.5\ kN.m$$

Step 2: Finding the rotations (ψ for each member) :

$$\psi_{AB} = \frac{\Delta}{l} = \frac{0}{6} = 0 \quad , \quad \psi_{BC} = \frac{\Delta}{l} = \frac{0}{9} = 0$$

Step 3: Finding the stiffness factors (k for each member) :

$$k_{AB} = \frac{I}{l} = \frac{I}{6} \quad , \quad k_{BC} = \frac{I}{l} = \frac{I}{9}$$

Step 4: Applying the slope – deflection equation for each member :

For the member AB : (Basic equation)

$$M_N = 2Ek[2\theta_N + \theta_F - 3\psi] + FEM_N \quad \text{and}$$
$$M_F = 2Ek[2\theta_F + \theta_N - 3\psi] + FEM_F$$

$$M_{AB} = 2Ek[2\theta_A + \theta_B - 3\psi] + FEM_{AB}$$
$$M_{AB} = (2EI/6)[\theta_B] = 0.333EI\theta_B \qquad (A1)$$
$$M_{BA} = 2Ek[2\theta_B + \theta_A - 3\psi] + FEM_{BA}$$
$$M_{BA} = (2EI/6)[2\theta_B] = 0.667EI\theta_B \qquad (A2)$$

For the member BC : (Basic equation)

$$M_N = 2Ek[2\theta_N + \theta_F - 3\psi] + FEM_N \quad \text{and}$$
$$M_F = 2Ek[2\theta_F + \theta_N - 3\psi] + FEM_F$$
$$M_{BC} = 2Ek[2\theta_B + \theta_C - 3\psi] + FEM_{BC}$$
$$M_{BC} = (2EI/9)[2\theta_B + \theta_C] - 67.5$$
$$M_{BC} = 0.444EI\theta_B + 0.222EI\theta_C - 67.5 \qquad (A3)$$
$$M_{CB} = 2Ek[2\theta_C + \theta_B - 3\psi] + FEM_{CB}$$
$$M_{CB} = (2EI/9)[2\theta_C + \theta_B] + 67.5$$
$$M_{CB} = 0.222EI\theta_B + 0.444EI\theta_C + 67.5 \qquad (A4)$$

Step 5: Applying the equilibrium equations for the joints B and C :

$$M_{BA} + M_{BC} = 0 \qquad (A5)$$
$$M_{CB} = 120 \qquad (A6)$$

Step 6: Solving the equations *(A1)* through *(A6)* simultaneously yields :

$$M_{AB} = 13.7\ kN.m$$
$$M_{BA} = 27.5\ kN.m$$
$$M_{BC} = -27.5\ kN.m$$

B. Solution II (By using the new modified slope – deflection equation – presented in this paper) :

The solution steps by using the new modified slope – deflection equation will be as follows :

Step 1: Finding the fixed end moments :

$$FEM_{AB} = 0$$
$$FEM_{BA} = 0$$
$$FEM_{BC} = -\frac{wl^2}{8} = -\frac{10(9)^2}{8} = -101.25\ kN.m$$

Step 2: Finding the rotations (ψ for each member) :

$$\psi_{AB} = \frac{\Delta}{l} = \frac{0}{6} = 0 \quad , \quad \psi_{BC} = \frac{\Delta}{l} = \frac{0}{9} = 0$$

Step 3: Finding the stiffness factors (k for each member) :

$$k_{AB} = \frac{I}{l} = \frac{I}{6} \quad , \quad k_{BC} = \frac{I}{l} = \frac{I}{9}$$

To specify the value of COM, since the joint B is intermediate roller, then it is considered to be as "fixed end", thus the value of COM will be $M/2$.

We have $M = 120\ kN.m$

Then $COM = M/2 = 120/2 = 60\ kN.m$

Step 4: Applying the slope – deflection equation for each member :

For the member AB: (Basic equation)

$$M_N = 2Ek[2\theta_N + \theta_F - 3\psi] + FEM_N \quad \text{and}$$
$$M_F = 2Ek[2\theta_F + \theta_N - 3\psi] + FEM_F$$
$$M_{AB} = 2Ek[2\theta_A + \theta_B - 3\psi] + FEM_{AB}$$
$$M_{AB} = (2EI/6)[\theta_B] = 0.333EI\theta_B \qquad (B1)$$
$$M_{BA} = 2Ek[2\theta_B + \theta_A - 3\psi] + FEM_{BA}$$
$$M_{BA} = (2EI/6)[2\theta_B] = 0.667EI\theta_B \qquad (B2)$$

For the member BC : **(New modified equation)**

$$M_N = 3Ek[\theta_N - \psi] + FEM_N + COM$$
$$M_{BC} = 3Ek[\theta_B - \psi] + FEM_{BC} + COM$$
$$M_{BC} = (3EI/9)[\theta_B] - 101.25 + 60$$
$$M_{BC} = 0.333EI\theta_B - 41.25 \qquad (B3)$$

Step 5: Applying the equilibrium equations for the joints B and C :

$$M_{BA} + M_{BC} = 0 \qquad (B4)$$
$$M_{CB} = 120 \qquad (B5)$$

Step 6: Solving the equations *(B1)* through *(B5)* simultaneously yields :

$$M_{AB} = 13.7\ kN.m$$
$$M_{BA} = 27.5\ kN.m$$
$$M_{BC} = -27.5\ kN.m$$

It can be clearly noticed that the two solutions give the same results, but the solution by using the new modified slope – deflection equation (*solution II*) was shorter and simpler.

V. Conclusions

The conclusions for this work can be summarized as follows:

1- The new modified slope – deflection equation reduces the computations for the analysis of beams and frames that have end spans supported by pin or roller supports at their ends and having an external moments at these ends.

2- The new modified slope – deflection equation reduces the computations for the analysis of beams and frames that have pin or roller supports with overhang cantilever ends.

3- The old modified slope – deflection equation can be replaced by the new modified slope – deflection equation since the cases that are solved by the old modified equation can also be solved easily by the new modified equation.

References

[1] R. C. Hibbeler, "Structural Analysis" , 8th edition, Pearson Prentice Hall, 2012.

[2] Aslam Qassimali , "Structural Analysis" , 4th edition, Cengage Learning, 2011.

[3] Alan Williams, "Structural Analysis, in Theory and Practice" , International Codes Council, 2009.

Muhammed Abbas Husain

- M.Sc. Civil Engineering / Structures – Tikrit University , Iraq, 1999.
- Lecturer in CEE department / College of Engineering /University of Samarra, Iraq.