Slip effects on the flow of a Carreau fluid through a porous medium in a planar channel under the effect of a magnetic field with peristalsis

B. Swaroopa and Prof. K. Ramakrishna Prasad

Abstract— **In this paper, the effects slip on the peristaltic flow of a Carreau fluid through a porous medium in a two dimensional channel under the assumptions of low Reynolds number and long wavelength is investigated. The flow is investigated in a wave frame of reference moving with velocity of the wave. The perturbation series in the Weissenberg number was used to obtain explicit forms for velocity field, pressure gradient per one wavelength. The effects of various pertinent parameters on the pressure gradient and pumping characteristics are discussed through graphs in detail.**

Index Terms—*Carreau fluid; Darcy number; MHD; Slip effects*

I. Introduction

Peristalsis is a series of wave-like muscle contractions that moves food to different processing stations in the digestive tract. This principle is used in designing the roller pumps which are useful in pumping machinery. For instance biomechanical pumps are fabricated to save blood or similar fluids from any possible contaminations arising out of contact with the pumping machinery while pumping the fluid. Since many physiological fluids behave like non-Newtonian in nature. Many authors have been studied the analysis of the mechanisms for peristaltic transport of non-Newtonian fluids [1-4]. Carreau fluid model is a four parameter model. The peristaltic transport of Carreau fluid by considering different flow models were developed by [5-8].

Specifically, the non-Newtonian fluids in the presence of a magnetic field are very useful in magneto-therapy. The controlled application of low intensity and frequency pulsing magnetic fields modify the cell and tissue behavior. Moreover, the non-invasive radiological test that uses a magnetic field (not radiation) to evaluate organs in abdomen prior to surgery in the small intestine (but not always). Hence magnetically susceptible of chyme can be satisfied from the heat generated by magnetic field or the ions contained in the chyme. The peristaltic flows of magneto hydrodynamic (MHD) fluid have been studied by [9-12].

The investigations of blood flow through arteries are of considerable importance in various cardiovascular diseases particularly arteriosclerosis. In some pathological situations, the distribution of fatty cholesterol and artery clogging blood clots in the lumen of coronary artery can be considered as equivalent to a porous medium. Reference [13] has studied the peristaltic mechanism of a Newtonian fluid through a porous medium. Reference [14] has investigated the MHD peristaltic flow of a porous medium in an asymmetric channel with heat transfer. Reference [15] have studied the Peristaltic motion of a Carreau fluid through a porous medium in a channel under the effect of a magnetic field**.**

Also flows with slip would be use full for problems in engineering, for example flows through pipe in which chemical reactions occur at the walls, two phase flow in porous slider bearings. The initial work on slip boundary condition on the peristaltic flow of a Maxwell fluid in a channel was discussed by [16]. The effects of slip and non-Newtonian parameters on the peristaltic flow of a third grade fluid in a circular cylindrical tube were investigated by [17]. Effects of slip and induced magnetic field on the peristaltic flow of pseudoplastic fluid were analyzed by [18]. Recently, [19] have investigated the slip effects on the peristaltic transport of a Jeffrey fluid through a porous medium in an asymmetric channel under the effect magnetic field.

In view of these, we studied the effects slip on the peristaltic flow of a Carreau fluid through a porous medium in a two dimensional channel under the assumptions of low Reynolds number and long wavelength. The flow is investigated in a wave frame of reference moving with velocity of the wave. The perturbation series in the Weissenberg number ($We < 1$) was used to obtain explicit forms for velocity field, pressure gradient per one wavelength. The effects of various pertinent parameters on the pressure gradient and pumping characteristics are discussed through graphs in detail.

II. Mathematical Formulation

We consider the flow of an incompressible Carreau fluid through a porous medium in a two dimensional planar channel with flexible walls. It is assumed that the progressive sinusoidal waves propagate along the walls of the channel. The fluid subjected to a constant transverse magnetic field. Induced magnetic field, external electric field, electric field due to polarization of charges, heat due to viscous and joule dissipation are neglected. The equation of the channel wall is given by

$$Y = H(X,t) = a + b\cos\frac{2\pi}{\lambda}(X - ct), \tag{1}$$

where b, λ, c and a are amplitude, wave length, phase speed of the wave, mean-half width of the channel respectively, t is the time and (X, Y) are the Cartesian co-ordinates. Fig. 1 represents the physical model of the channel.

Manuscript received April 22, 2015.

B. Swaroopa, Department of Mathematics, Govt. Degree College, Rajampet, Andhra Pradesh-516115, India

Prof. K. Ramakrishna Prasad, Department of Mathematics, S. V. University, Tirupati-517503, Andhra Pradesh, India

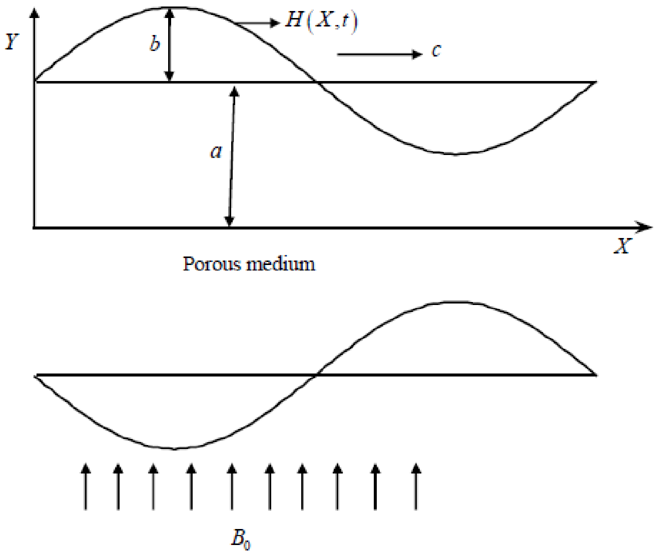

Fig. 1. The physical model

We introduce a wave frame of reference (x, y) moving with the velocity c in which the motion becomes independent of time when the channel length is an integral multiple of the wave length and the pressure difference at the ends of the channel is a constant. The transformation from the fixed frame of reference (X,Y) to the wave frame of reference (x, y) is given by

$x = X - ct,\ \ y = Y,\ \ u = U - c,\ \ v = V,\ \ p(x) = P(X,t)$. (2)

where (u,v) and (U,V) are the velocity components, p and P are pressures in the wave and fixed frames of reference respectively.

The constitute equation for a Carreau fluid (given in [20]) is

$$\tau = -\left[\eta_\infty + (\eta_0 - \eta_\infty)\left(1 + (\Gamma\dot{\gamma})^2\right)^{\frac{n-1}{2}}\right]\dot{\gamma} \tag{3}$$

where τ is the extra stress tensor, η_∞ is the infinite shear rate viscosity, η_0 is the zero shear rate viscosity, Γ is the time constant, n is the dimensionless power-law index and $\dot{\gamma}$ is defined as

$$\dot{\gamma} = \sqrt{\frac{1}{2}\sum_i\sum_j \dot{\gamma}_{ij}\dot{\gamma}_{ji}} = \sqrt{\frac{1}{2}\pi} \tag{4}$$

here π is the second invariant of strain-rate tensor. We consider in the constitutive equation (3) the case for which $\eta_\infty = 0$ and so we can write

$$\tau = -\eta_0\left(1 + (\Gamma\dot{\gamma})^2\right)^{\frac{n-1}{2}}\dot{\gamma}\,. \tag{5}$$

The Carreau model reduces to Newtonian model for $n = 1$ (or) $\Gamma = 0$.

The equations governing the flow in the wave frame of reference are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{6}$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} - \frac{\partial \tau_{xx}}{\partial x} - \frac{\partial \tau_{yx}}{\partial y} - \left(\sigma B_0^2 + \frac{1}{k}\right)(u+1) \tag{7}$$

$$\rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} - \frac{\partial \tau_{xy}}{\partial x} - \frac{\partial \tau_{yy}}{\partial y} - \frac{1}{k}v \tag{8}$$

where ρ is the density, k is the permeability of the porous medium, σ is the electrical conductivity and B_0 is constant transverse magnetic field.

Introducing the non-dimensional variable defined by

$$\bar{x} = \frac{x}{\lambda},\ \ \bar{y} = \frac{y}{a},\ \ \bar{u} = \frac{u}{c},\ \ \bar{v} = \frac{v}{c\delta},\ \ \delta = \frac{a}{\lambda}, \bar{p} = \frac{pa^2}{\eta_0 c\lambda},\ \ h = \frac{H}{a},$$

$$\bar{t} = \frac{ct}{\lambda},\ \ \bar{\tau}_{xx} = \frac{\lambda}{\eta_0 c}\tau_{xx}, \bar{\tau}_{xy} = \frac{a}{\eta_0 c}\tau_{xy},\ \ \bar{\tau}_{yy} = \frac{a}{\eta_0 c}\tau_{yy},\ \ \mathrm{Re} = \frac{\rho a c}{\eta_0},$$

$$We = \frac{\Gamma c}{a},\ \ \bar{\dot{\gamma}} = \frac{\dot{\gamma} a}{c},\ \ \bar{q} = \frac{q}{ac}, Da = \frac{k}{a^2} \tag{9}$$

where Re is the Reynolds number and δ is the wave number, into the equations (6) – (8) (dropping bars), we get

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{10}$$

$$\mathrm{Re}\,\delta\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} - \delta^2\frac{\partial \tau_{xx}}{\partial x} - \frac{\partial \tau_{yx}}{\partial y} - \left(\frac{1}{Da} + M^2\right)(u+1) \tag{11}$$

$$\mathrm{Re}\,\delta^3\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} - \delta^2\frac{\partial \tau_{xy}}{\partial x} - \delta\frac{\partial \tau_{yy}}{\partial y} - \frac{\delta^2}{Da}v\,. \tag{12}$$

where $\tau_{xx} = -2\left[1 + \left(\frac{n-1}{2}\right)We^2\dot{\gamma}^2\right]\frac{\partial u}{\partial x}$,

$$\tau_{xy} = -\left[1 + \left(\frac{n-1}{2}\right)We^2\dot{\gamma}^2\right]\left(\frac{\partial u}{\partial y} + \delta^2\frac{\partial v}{\partial x}\right),$$

$$\tau_{yy} = -2\delta\left[1 + \left(\frac{n-1}{2}\right)We^2\dot{\gamma}^2\right]\frac{\partial v}{\partial y},$$

$$\dot{\gamma} = \left[2\delta^2\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y} - \delta^2\frac{\partial v}{\partial x}\right)^2 + 2\delta^2\left(\frac{\partial v}{\partial y}\right)^2\right]^{\frac{1}{2}},$$

and $M = a\mu_e H_0\sqrt{\frac{\sigma}{\eta_0}}$ is the Hartman number and $Da = \frac{k}{a^2}$ is the Darcy number.

Under lubrication approach, neglecting the terms of order δ and Re, we get

$$\frac{\partial p}{\partial x} = \frac{\partial}{\partial y}\left[1 + \left(\frac{n-1}{2}\right)We^2\left(\frac{\partial u}{\partial y}\right)^2\right]\frac{\partial u}{\partial y} - N^2(u+1) \tag{13}$$

$$\frac{\partial p}{\partial y} = 0 \tag{14}$$

here $N = \sqrt{M^2 + \frac{1}{Da}}$.

The corresponding boundary conditions in wave frame of reference are given by

$$u + \beta\left[\frac{\partial u}{\partial y} + \left(\frac{n-1}{2}\right)We^2\left(\frac{\partial u}{\partial y}\right)^3\right] = -1 \text{ at } y = h, \tag{15}$$

$$\frac{\partial u}{\partial y} = 0 \quad \text{at } y = 0\,. \tag{16}$$

From Equations (13) and (14) it is seen that p is independent of y. So that (13) can be rewritten as

$$\frac{dp}{dx}=\frac{\partial}{\partial y}\left\{\left[1+\left(\frac{n-1}{2}\right)We^2\left(\frac{\partial u}{\partial y}\right)^2\right]\frac{\partial u}{\partial y}\right\}-N^2(u+1), \quad (17)$$

The volume flow rate q in a wave frame of reference is given by

$$q=\int_0^h u dy. \quad (18)$$

The instantaneous flux $Q(X,t)$ in the laboratory frame is

$$Q(x,t)=\int_0^h u dy=\int_0^h (u+1)dy=q+h. \quad (19)$$

The time average flux over one period $T\left(=\frac{\lambda}{c}\right)$ of the peristaltic wave is

$$\bar{Q}=\frac{1}{T}\int_0^T Qdt=\int_0^1 (q+h)dx=q+1. \quad (20)$$

III. SOLUTION

Since Eq. (2.17) is non-linear differential equation, it is not possible to obtain closed form solution. So, we seek a perturbation solution by considering Wiessenberg number We as a small parameter. For perturbation solution, we expand u, q and p as

$$u=u_0+We^2u_1+O(We^4) \quad (21)$$

$$q=q_0+We^2q_1+O(We^4) \quad (22)$$

$$\frac{dp}{dx}=\frac{dp_0}{dx}+We^2\frac{dp_1}{dx}+O(We^4) \quad (23)$$

Substituting these equations in (17) and in boundary conditions (15) and (16), we get

3.1 System of order We^0

$$\frac{dp_0}{dx}=\frac{\partial^2 u_0}{\partial y^2}-N^2(u_0+1). \quad (24)$$

The boundary conditions are

$$u_0+\beta\frac{\partial u_0}{\partial y}=-1 \quad \text{at} \quad y=h \quad (25)$$

$$\frac{\partial u_0}{\partial y}=0 \quad \text{at} \quad y=0 \quad (26)$$

3.2 System of order We^2

$$\frac{dp_1}{dx}=\frac{\partial^2 u_1}{\partial y^2}+\left(\frac{n-1}{2}\right)\frac{\partial}{\partial y}\left[\left(\frac{\partial u_0}{\partial y}\right)^3\right]-N^2u_1 \quad (27)$$

The boundary conditions are

$$u_1+\beta\left[\frac{\partial u_1}{\partial y}+\left(\frac{n-1}{2}\right)\left(\frac{\partial u_0}{\partial y}\right)^3\right]=0 \quad \text{at} \quad y=h \quad (28)$$

$$\frac{\partial u_1}{\partial y}=0 \quad \text{at} \quad y=0 \quad (29)$$

3.3. Solution for system of order We^0

Solving Eq. (24) and then using the boundary condition equations (25) and (26), we get

$$u_0=\frac{1}{c_1N^2}\frac{dp_0}{dx}(\cosh Ny-c_1)-1 \quad (30)$$

where $c_1=\cosh Nh+\beta\sinh Nh$.

and the volume flow rate q_0 is given by

$$q_0=\int_0^h u_0 dy=\frac{1}{c_1N^3}\frac{dp_0}{dx}[\sinh Nh-Nhc_1]-h \quad (31)$$

From Eq. (3.11), we get

$$\frac{dp_0}{dx}=\frac{c_1(q_0+h)N^3}{\sinh Nh-Nhc_1}. \quad (32)$$

3.4 Solution for system of order We^2

Solving Eq. (3.7) using the Eq. (3.10) and the boundary conditions (3.8) and (3.9), we get

$$u_1=\frac{1}{c_1N^2}\frac{dp_1}{dx}[\cosh Ny-c_1]+\frac{3(n-1)}{64N^4c_1^4}\left(\frac{dp_0}{dx}\right)^3\begin{bmatrix}c_5\cosh Ny-c_1\cosh 3Ny\\+4c_1Ny\sinh Ny\end{bmatrix} \quad (33)$$

where $c_2=\cosh 3Nh-4Nh\sinh Nh$,

$c_3=3N\sinh 3Nh-4N\sinh Nh-4N^2h\cosh Nh$,

$c_4=\frac{32}{3}N\sinh^3 Nh$ and $c_5=c_2+\beta c_3-\beta c_4$.

and the volume flow rate q_1 is given by

$$q_1=\int_0^h u_1 dy=\frac{1}{c_1c^3}\frac{dp_1}{dx}[\sinh Nh-Nhc_1]+\frac{(n-1)c_6}{64N^5c_1^4}\left(\frac{dp_0}{dx}\right)^3 \quad (34)$$

where

$c_6=3c_5\sinh Nh-c_1\sinh 3Nh+12Nhc_1\cosh Nh-12c_1\sinh Nh$.

From Eq. (3.14) and Eq. (3.12), we have

$$\frac{dp_1}{dx}=\frac{q_1N^3c_1}{(\sinh Nh-Nhc_1)}-\frac{(n-1)c_6N^7(q_0+h)^3}{64(\sinh Nh-Nhc_1)^4} \quad (35)$$

Substituting from Equations (32) and (35) into (23) and using the relation $\frac{dp_0}{dx}=\frac{dp}{dx}-We^2\frac{dp_1}{dx}$ and neglecting terms greater than $O(We^2)$, we get

$$\frac{dp}{dx}=\left(\frac{N^3c_1(q+h)}{(\sin Nh-Nhc_1)}-We^2\frac{c_6(n-1)N^7(q+h)^3}{64(\sin Nh-Nhc_1)^4}\right) \quad (36)$$

The dimensionless pressure rise per one wavelength in the wave frame is defined as

$$\Delta p=\int_0^1\frac{dp}{dx}dx \quad (37)$$

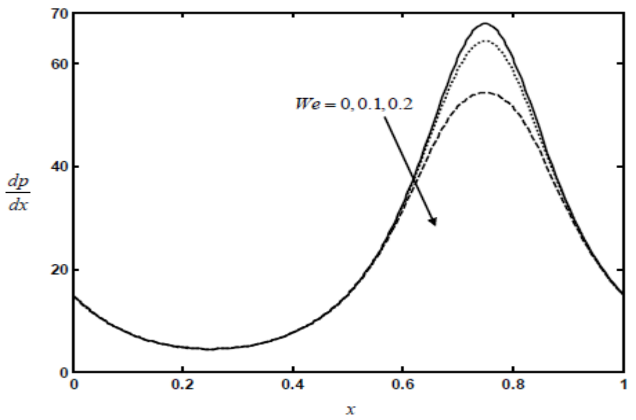

Fig. 2. The variation of axial pressure gradient $\frac{dp}{dx}$ with We for $\phi = 0.6$, $Da = 0.1$, $M = 1$, $\beta = 0.1$ and $n = 0.398$.

IV. Results And Discussions

In order to see the effects of various parameters like Wiessenberg number We, power-law index n, slip parameter β, Hartmann number M, Darcy number Da, amplitude ratio ϕ on the axial pressure gradient $\frac{dp}{dx}$ we plotted Figs. 2-7. From Fig. 2, it is found that the axial pressure gradient $\frac{dp}{dx}$ decreases with increasing We. From Fig. 3, it is noticed that the axial pressure gradient $\frac{dp}{dx}$ increases with an increase in n. From Fig. 4, it is observed that the axial pressure gradient $\frac{dp}{dx}$ decreases with increasing β. From Fig. 5, it is found that the axial pressure gradient $\frac{dp}{dx}$ increases on increasing M. From Fig. 6, it is noted that the axial pressure gradient $\frac{dp}{dx}$ decreases with an increase in Da. From Fig. 7 it is noticed that, the axial pressure gradient $\frac{dp}{dx}$ increases with increasing ϕ.

In order to see the effects of various parameters like Wiessenberg number We, power-law index n, slip parameter β, Hartmann number M, Darcy number Da, amplitude ratio ϕ on the the time-averaged volume flow rate $\bar{Q}$ we plotted Figs. 8-13. From Fig. 8, it is observed that the time-averaged volume flow rate $\bar{Q}$ decreases with increasing We in the pumping region $(\Delta p > 0)$, while it increases with increasing We in both the free pumping $(\Delta p = 0)$ and co-pumping $(\Delta p < 0)$ regions. From Fig. 9, it is found that, the time-averaged volume flow rate $\bar{Q}$ increases with increasing n in the pumping region, while it decreases with increasing n in both the free pumping and co pumping regions. Moreover, it is seen that the pumping is less for Carreau fluid than that of Newtonian fluid $(n \to 1)$. From Fig. 10, it is found that the time-averaged volume flow rate $\bar{Q}$ decreases with increasing β in both the pumping and free pumping regions, while it increases with increasing β in the co-pumping region for chosen $\Delta p(<0)$. From Fig. 11, it is observed that the time-averaged volume flow rate $\bar{Q}$ increases with increasing M in the pumping region, while it decreases with increasing M in both the free pumping and co pumping regions. Fig. 12, it is found that the time-averaged volume flow rate $\bar{Q}$ decreases with increasing Da in the pumping region, while it increases with increasing Da in both the free pumping and co pumping regions. From Fig. 13, it is noted that, the time-averaged volume flow rate $\bar{Q}$ increases with increasing ϕ in both the pumping and free pumping regions, while it decreases with increasing ϕ in the co- pumping region for chosen $\Delta p(<0)$.

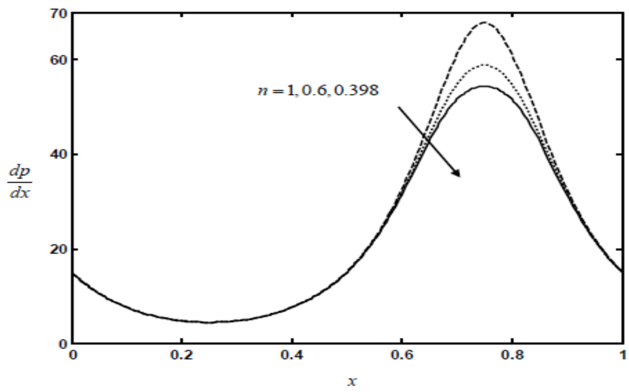

Fig. 3. The variation of axial pressure gradient $\frac{dp}{dx}$ with n for $\phi = 0.6$, $Da = 0.1$, $M = 1$, $\beta = 0.1$ and $We = 0.1$.

V. Conclusion

In this paper, we studied the influence of slip on the peristaltic flow of a Carreau fluid through a porous medium in a planar channel with the effect of a magnetic field under the assumptions of long wavelength and low-Reynolds number assumptions. It is observed that, the axial pressure gradient and time averaged flux in the pumping region increases with increasing n, M and ϕ, whereas they decreases with increasing We, β and Da.

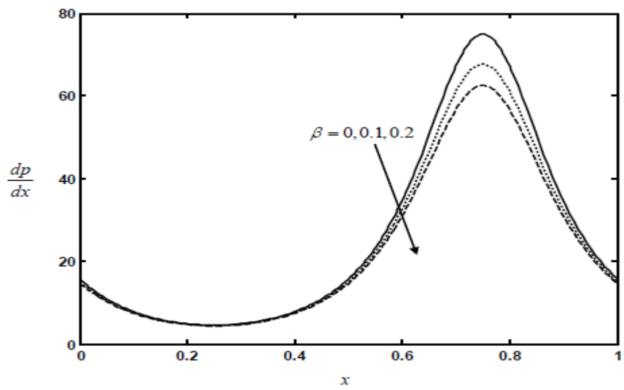

Fig. 4. The variation of axial pressure gradient $\frac{dp}{dx}$ with β for $\phi = 0.6$, $Da = 0.1$, $M = 1$, $We = 0.1$ and $n = 0.398$.

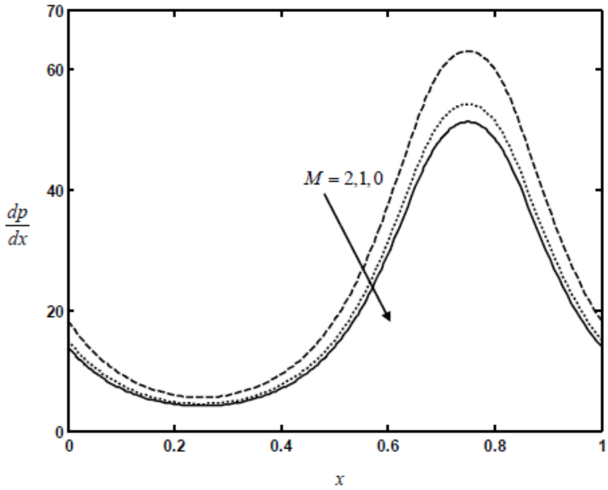

Fig. 5. The variation of axial pressure gradient $\frac{dp}{dx}$ with M for $\phi = 0.6$, $Da = 0.1$, $We = 0.1$, $\beta = 0.1$ and $n = 0.398$.

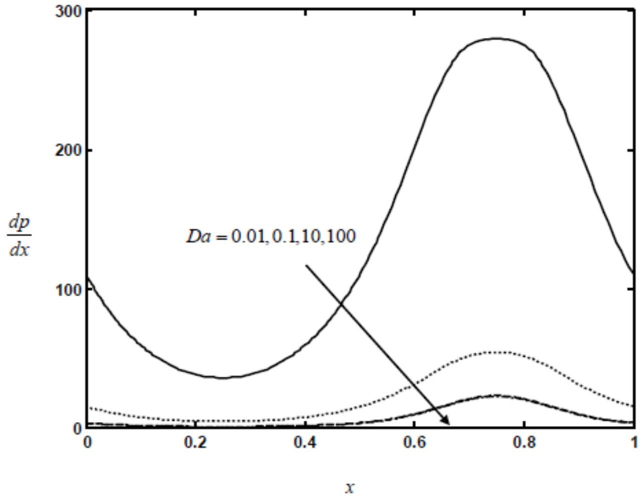

Fig. 6. The variation of axial pressure gradient $\frac{dp}{dx}$ with Da for $\phi = 0.6$, $M = 1$, $We = 0.1$, $\beta = 0.1$ and $n = 0.398$.

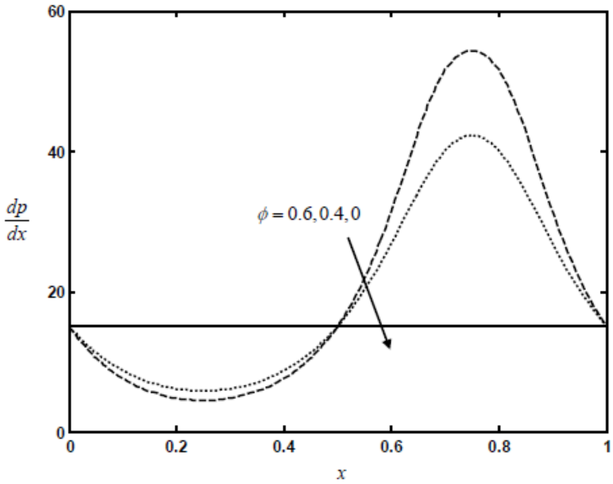

Fig. 7. The variation of axial pressure gradient $\frac{dp}{dx}$ with ϕ for $We = 0.1$, $M = 1$, $\beta = 0.1$ and $n = 0.398$.

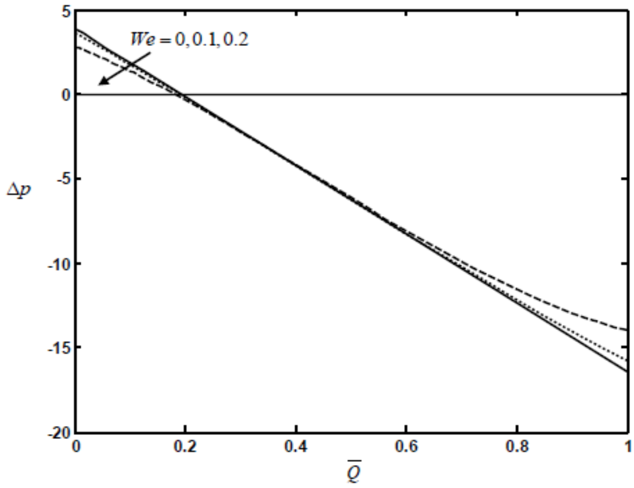

Fig. 8 The variation of pressure rise Δp with time-averaged volume flow rate $\overline{Q}$ for different values of We with $\phi = 0.6$, $M = 1$, $\beta = 0.1$, $Da = 0.1$ and $n = 0.398$.

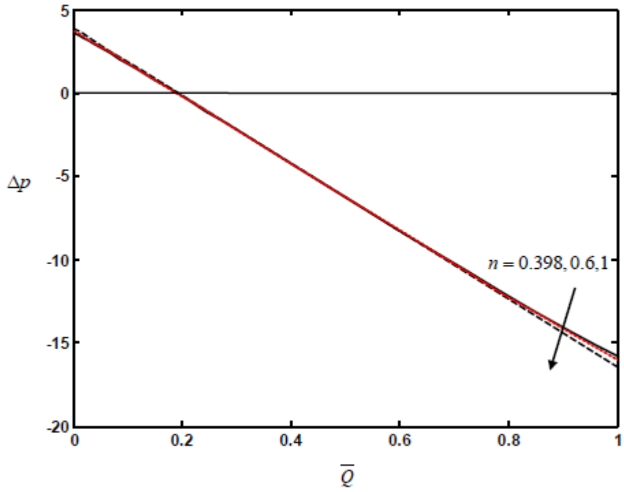

Fig. 9. The variation of pressure rise Δp with time-averaged volume flow rate $\overline{Q}$ for different values of n with $\phi = 0.6$, $M = 1$, $\beta = 0.1$, $Da = 0.1$ and $We = 0.1$.

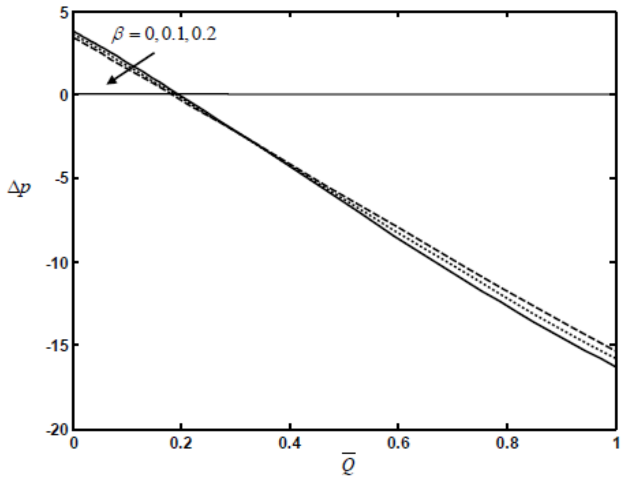

Fig. 10. The variation of pressure rise Δp with time-averaged volume flow rate $\overline{Q}$ for different values of β with $\phi = 0.6$, $M = 1$, $We = 0.1$ and $n = 0.398$.

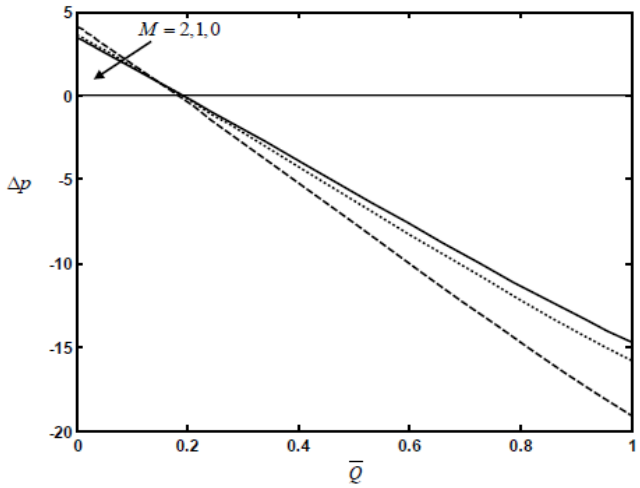

Fig. 11. The variation of pressure rise Δp with time-averaged volume flow rate $\bar{Q}$ for different values of M with $\phi = 0.6$, $We = 0.1$, $\beta = 0.1$, $Da = 0.1$ and $n = 0.398$.

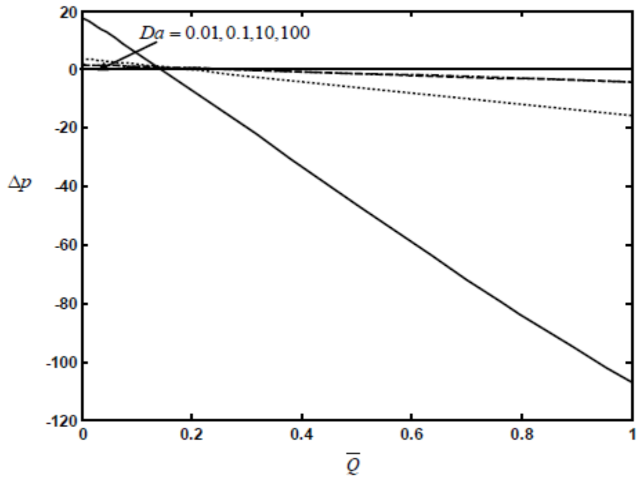

Fig. 12. The variation of pressure rise Δp with time-averaged volume flow rate $\bar{Q}$ for different values of Da with $\phi = 0.6$, $We = 0.1$, $\beta = 0.1$, $M = 1$ and $n = 0.398$.

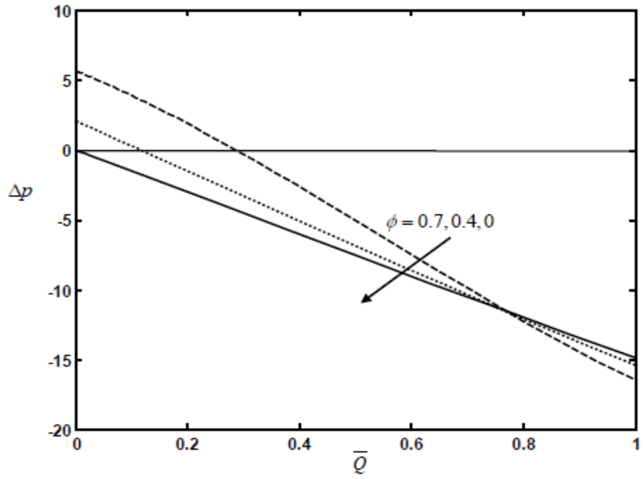

Fig. 13. The variation of pressure rise Δp with time-averaged volume flow rate $\bar{Q}$ for different values of We with $\phi = 0.6$, $M = 1$, $\beta = 0.1$, $Da = 0.1$ and $n = 0.4$.

References

[1] G. Radhakrishnamacharya, “Long wavelength approximation to peristaltic motion of a power law fluid,” *Rheol. Acta*., vol. 21, 1982, pp. 30-35.

[2] L.M. Srivastava, and V.P. Srivastava, “Peristaltic transport of a non – Newtonian fluid: application to the vas deferens and small intestine,” *Annals Biomed. Engng*., vol. 13, 1985, pp. 137-153.

[3] M.V. Subba Reddy, A. Ramachandra Rao and S. Sreenadh, “Peristaltic motion of a power law fluid in an asymmetric channel,” *Int. J. Non-Linear Mech*., 42, 2007, pp. 1153-1161.

[4] M. V. Subba Reddy and C. Nadhamuni Reddy, “Peristaltic flow of a non-Newtonian fluid through a porous medium in a tube with variable viscosity using Adomian decomposition method,” *Int. Review of Appl. Engng. Res*., vol. 4, no. 1, 2014pp. 137-146.

[5] A. M. El Misery, E. F. El Shehawey and A. A. Hakeem, “Peristaltic motion of an incompressible generalized Newtonian fluid in a planar channel,” *J. Phys. Soc. Jpn*., vol. 65, 1996, pp. 3524-3529.

[6] E.F. El Shehawey, A. M. EL Misery and A. A. Hakeem, “Peristaltic motion of generalized Newtonian fluid in a non-uniform channel,” *J. Phys. Soc. Jpn*., vol. 67, 1998, pp. 434-440.

[7] N. Abd El Hakeem and A.E.M. El Misiery,” Effects of an endoscope and generalised Newtonian Fluid on peristaltic motion,” *Appl. Math. Comput*., vol. 128, 2002, pp. 19-35.

[8] N. Ali and T. Hayat, “Peristaltic motion of a Carreau fluid in an asymmetric channel,” *Appl. Math. Comput*., vol. 194, 2007, pp. 535-552.

[9] V.K. Stud, G.S. Sephon and R.K. Mishra, “Pumping action on flow by a magnetic field,” *Bull. Math. Biol*. vol. 39, 1977, pp. 385-390.

[10] L. M. Srivastava, and R.P. Agrawal, “Oscillating flow of a conducting fluid with a suspension of spherical particles,” *J. Appl. Mech*. Vol. 47, 1980, pp. 196-199.

[11] H.L. Agrawal, and B. Anwaruddin, ‘Peristaltic flow of blood in a branch,” *Ranchi University Math. J.* vol. 15, 1984, pp. 111-121.

[12] Kh.S. Mekheimer, “Peristaltic flow of blood under effect of a magnetic field in a non uniform channel,” *Appl. Math. Comput*., vol. 153, 2004, pp. 763-777.

[13] E.F. El Shehawey, and S.Z.A. Husseny, “Effects of porous boundaries on peristaltic transport through a porous medium,” *Acta Mechanica*, vol. 143, 2000, pp. 165-177.

[14] T. Hayat, M. Umar Qureshi, and Q. Hussain, “Effect of heat transfer on the peristaltic flow of an electrically conducting fluid in a porous space,” *Applied Mathematical Modelling*, vol. 33, 2009, pp. 1862–1873.

[15] M. Sudhakar Reddy, M.V. Subba Reddy and S. Ramakrishna, “Peristaltic motion of a carreau fluid through a porous medium in a channel under the effect of a magnetic field,” *Far East Journal of Applied Mathematics*, vol. 35, 2009, pp. 141-158.

[16] E.F. El-Shehawey, N.T. El-Dabe, and I. M. El-Desoky, “Slip effects on the Peristaltic flow of a Non-Newtonian Maxwellian Fluid,” *Acta Mech*., vol. 186, 2006, pp. 141-159.

[17] N. Ali, Y. Wang, T. Hayat, and M. Oberlack, “Slip effects on the peristaltic flow of a third grade fluid in a circular cylindrical tube,” *J. Appl. Mech*., vol. 76, 2009, pp. 1-10.

[18] S. Noreen, T. Hayat, and A. Alsaedi, “Study of slip and induced magnetic field on the peristaltic flow of pseudoplastic fluid,” *International Journal of Physical Sciences*, vol. 6, no.(36), 2011, pp. 8018-8026.

[19] M.V. Subba Reddy, B. Jayarami Reddy, N. Nagendra. and B. Swaroopa, “Slip effects on the peristaltic motion of a Jeffrey fluid through a porous medium in an asymmetric channel under the effect magnetic field,” *J. Appl. Math. and Fluid Mechanics*,vol. 4, 2012, pp. 59-72.

[20] R.B. Bird, R.C. Armstrong, and Hassager, “*Dynamics of Polymeric Liquids*,” John Wiley & Sons., New York, (1977) Vol. 1, Chap. 5, p. 210.