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Generalized Class of Sakaguchi Functions in Conic Region

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***Abstract*— In this paper, we use the concept of Sakaguchi type functions, Janowski functions and the conic regions are combined to define a class of functions in a new interesting conic domain. We prove coefficient inequalities and inclusion results.**

***Index Terms*— Analytic functions, Sakaguchi type functions, Conic domains, Janowski functions, $k-$ Starlike functions, $k-$ Uniformly convex functions.**

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I. INTRODUCTION

Let A be the class of functions of form

$$f(z)=z+\sum_{n=2}^{\infty}a_n z^n, \quad (1)$$

Which are analytic in the open unit disk

$U=\{z:z\in\mathbb{C}\ and\ |z|<1\}$, with normalization

$f(0)=0\ and\ f^{\prime}(0)=1.$

Consider the conic region $\Omega_k, k\geq$ given by

$$\Omega_k=\left\{u+iv:u>k\sqrt{(u-1)^2+v^2}\right\}.$$

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This domain represents the right half plane for $k=0$, hyperbola for $0<k<1$, a parabola for $k=1$ and ellipse for $k>1$. The functions $p_k(z)$ play the role of external functions for these conic regions where

$$p_k(z)=\begin{cases}\dfrac{1+z}{1-z}, k=0\\ 1+\dfrac{2}{\pi^2}\left(\log\dfrac{1+\sqrt{z}}{1-\sqrt{z}}\right)^2, k=1.\\ 1+\dfrac{2}{1-k^2}\sinh^2\left[\left(\dfrac{2}{\pi}\arccos k\right)\arctan h\sqrt{z}\right], 0<k<1.\\ 1+\dfrac{2}{k^2-1}\sin\left[\dfrac{\pi}{2R(t)}\int_0^{\frac{u(z)}{\sqrt{t}}}\dfrac{1}{\sqrt{1-x^2}\sqrt{1-(tx)^2}}dx\right]+\dfrac{1}{k^2-1}, k>1,\end{cases} \quad (2)$$

Where $u(z)=\dfrac{z-\sqrt{1}}{1-\sqrt{tx}}, t\in(0,1), t\in U$ and z is chosen such that $k=\cosh\left(\dfrac{\pi R^{\prime}(t)}{4R(t)}\right), R(t)$ is the Legendre's complete elliptic integral of the first kind and $R^{\prime}(t)$ is complementary integral $R(t)$. $p_k(z)=1+\delta_k z+.....[9]$ where

$$\delta_k=\begin{cases}\dfrac{8(\arccos k)^2}{\pi^2(1-k^2)}, 0\leq k<1\\ \dfrac{8}{\pi^2}, k=1\\ \dfrac{\pi^2}{4(k^2-1)\sqrt{t(1+t)R^2(t)}}, k>1.\end{cases} \quad (3)$$

Using the concept of conic regions we define the following:

Definition 1.1. A function $f\in A$ is said to be the class $k-UB(\alpha,\beta,\gamma,s,t)$, for $k\geq 0,\alpha\leq 0,0\leq\beta,<1,0\leq\gamma<1s,t\in\mathbb{C}$ with $s\neq t$ if and only if

$$\Re\left(J(\alpha,\beta,\gamma,s,t,f(z))\right)>\left|J(\alpha,\beta,\gamma,s,t,f(z))-1\right|,$$

where

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$$J\left(\alpha,\beta,\gamma,s,t,f\left(z\right)\right)=\frac{1-\alpha}{1-\beta}\left(\frac{\left(s-t\right)zf'\left(z\right)}{f\left(sz\right)-f\left(tz\right)}-\beta\right)+\frac{\alpha}{1-\gamma}\left(\frac{\left(s-t\right)\left(zf'\left(z\right)\right)'}{\left(f\left(sz\right)-f\left(tz\right)\right)'}-\gamma\right).$$

Or equivalently

$$J\left(\alpha,\beta,\gamma,s,t,f\left(z\right)\right)\prec p_k\left(z\right),$$

where $p_k\left(z\right)$ is defined by (6).

This class generalizes various classes studied by Khalida Inayat Noor and Sarfraz Naeaz Malik in [6], Kanas and Winsiowska [3, 10], Shams and Kulkarni [4], Kanas [7], Mocaun [1], Goodman [5].

II. MAIN RESULTS

Theorem 2.1. A function $f\in A$ and of the form (1) is in the class $k-UB\left(\alpha,\beta,\gamma,s,t\right)$, if it satisfies the condition

$$\sum_{n=2}^{\infty}\Psi\left(k;\alpha,\beta,\gamma,s,t\right)<\left(1-\beta\right)\left(1-\gamma\right), \tag{4}$$

Where

$$\Psi_n\left(k;\alpha,\beta,\gamma,s,t\right)=\left(1-\beta\right)\left(1-\gamma\right)\sum_{j=2}^{n-1}\left(n+1-j\right)u_j\left(s,t\right).u_n+1-j\left(s,t\right)\left|a_ja_{n+1-j}\right|+$$
$$\left(k+1\right)\left|\left(1-\alpha\right)\left(1-\gamma\right)\left(1+u_n\left(s,t\right)\right)n-\left[\left(1-\gamma\right)+\alpha\left(\gamma-\beta\right)\right]\left(n+1\right)u_n\left(s,t\right)+\alpha\left(1-\beta\right)\left(n^2+u\left(n\right)\left(s,t\right)\right)\right|\left|a_n\right|$$
$$+\sum_{j=2}^{n-1}\left(k+1\right)\left|\left(j\left(1-\alpha\right)\left(1-\gamma\right)-u_n\left(s,t\right)\left[\left(1-\gamma\right)+\alpha\left(\gamma-\beta\right)\right]\right)\left(n+1-j\right)u\left(n+1-j\right)\left(s,t\right)a_ja_{n+1-j}\right|$$
$$+\sum_{j=2}^{n-1}\left(k+1\right)\left|\alpha\left(1-\beta\right)\left(n+1-j\right)^2u_j\left(s,t\right)a_ja_{n+1-j}\right|+\left(1-\beta\right)\left(1-\gamma\right)\left(n+1\right)u_n\left(s,t\right)\left|a_n\right|,$$

were $k\geq 0,\alpha\leq 0,0\leq\beta,<1,0\leq\gamma<1$ and $u_n\left(s,t\right)=\sum_{j=0}^{n-1}s^{n-j}t^{j-1}$

Proof. Assuming equation (8) holds, then it suffices to show that

$$k\left|J\left(\alpha,\beta,\gamma,s,t,f\left(z\right)\right)-1\right|-\Re\left(J\left(\alpha,\beta,\gamma,s,t,f\left(z\right)\right)-1\right)<1.$$

Now consider $\left|J\left(\alpha,\beta,\gamma,s,t,f\left(z\right)\right)-1\right|$, then

$$\left|\frac{1-\alpha}{1-\beta}\left(\frac{\left(s-t\right)zf'\left(z\right)}{f\left(sz\right)-f\left(tz\right)}-\beta\right)+\frac{\alpha}{1-\gamma}\left(\frac{\left(s-t\right)\left(zf'\left(z\right)\right)'}{\left(f\left(sz\right)-f\left(tz\right)\right)'}-\gamma\right)-1\right|$$

$$\left|\frac{\begin{array}{l}\left(1-\alpha\right)\left(1-\gamma\right)\left(s-t\right)zf'\left(z\right)\left(f\left(sz\right)-f\left(tz\right)\right)-\left[\left(1-\gamma\right)+\alpha\left(\gamma-\beta\right)\right]\left(f\left(sz\right)-f\left(tz\right)\right)\left(f\left(sz\right)-f\left(tz\right)\right)'+\\ \alpha\left(1-\beta\right)\left\{\left(s-t\right)^2\left(zf'\left(z\right)\right)'\left(f\left(sz\right)-f\left(tz\right)\right)\right\}\end{array}}{\left(1-\beta\right)\left(1-\gamma\right)\left(f\left(sz\right)-f\left(tz\right)\right)\left(f\left(sz\right)-f\left(tz\right)\right)'}\right| \tag{5}$$

Now from (1) and we get

$$(s-t)zf^{\prime}(z)\left(f(sz)-f(tz)\right)^{\prime}=(s-t)^{2}z\left[\sum_{n=0}^{\infty}na_{n}z^{n-1}\right]\left[\sum_{n=0}^{\infty}nu_{n}(s,t)a_{n}z^{n-1}\right],a_{0}=u_{0}(s,t),a_{1}=u_{1}(s,t)=1,$$

$$=(s-t)^{2}\frac{1}{z}\left[\sum_{n=0}^{\infty}na_{n}z^{n-1}\right]\left[\sum_{n=0}^{\infty}nu_{n}(s,t)a_{n}z^{n}\right]$$

$$=(s-t)^{2}\frac{1}{z}\sum_{n=0}^{\infty}\left[\sum_{j=0}^{n}j(n-j)u_{n-j}(s,t)a_{j}a_{n-j}\right]z^{n}$$

$$=\sum_{n=0}^{\infty}\left[\sum_{j=0}^{n}j(n-j)u_{n-j}(s,t)a_{j}a_{n-j}\right]z^{n-1}$$

$$=(s-t)^{2}\left\{z+\sum_{n=3}^{\infty}\left[\sum_{j=0}^{n}j(n-j)u_{n-j}(s,t)a_{j}a_{n-j}\right]z^{n-1}\right\}$$

$$=(s-t)^{2}\left\{z+\sum_{n=2}^{\infty}\left[\left(1+u_{n}(s,t)\right)na_{n}+\sum_{j=2}^{n-1}j(n+1-j)u_{n+1-j}(s,t)a_{j}a_{n+1-j}\right]z^{n}\right\}.$$

Similarly, we get

$$\left(f(sz)-f(tz)\right)\left(f(sz)-f(tz)\right)^{\prime}=(s-t)^{2}\left\{z+\sum_{n=2}^{\infty}\left[\begin{matrix}(n+1)u_{n}(s,t)a_{n}+\\ \sum_{j=2}^{n-1}(n+1-j)u_{j}(s,t).u_{n+1-j}(s,t)a_{j}a_{n+1-j}\end{matrix}\right]z^{n}\right\}\text{ and}$$

$$\left(f(sz)-f(tz)\right)\left(f(sz)-f(tz)\right)^{\prime}=(s-t)^{2}\left\{z+\sum_{n=2}^{\infty}\left[\left(n^{2}+u_{n}(s,t)\right)a_{n}+\sum_{j=2}^{n-1}(n+1-j)^{2}u_{j}(s,t)a_{j}a_{n+1-j}\right]z^{n}\right\},$$

Using the above equalities in (9), we get

$$\frac{\sum_{n=2}^{\infty}\left[(1-\alpha)(1-\gamma)\left(1+u_{n}(s,t)\right)n-\left[(1-\gamma)+\alpha(\gamma-\beta)\right](n+1)u_{n}(s,t)+\alpha(1-\beta)\left(n^{2}+u_{n}(s,t)\right)\right]a_{n}z^{n}}{(1-\beta)(1-\gamma)\left[z+\sum_{n=2}^{\infty}\left[(n+1)u_{n}(s,t)a_{n}+\sum_{j=2}^{n-1}(n+1-j)u_{j}(s,t).u_{n+1-j}(s,t)a_{j}a_{n+1-j}\right]z^{n}\right]}$$

$$+\frac{\sum_{n=2}^{\infty}\left[\sum_{j=2}^{n-1}\left(j(1-\alpha)(1-\gamma)u_{n+1-j}(s,t)\right)\right](n+1-j)a_{j}a_{n+1-j}z^{n}}{(1-\beta)(1-\gamma)\left[z+\sum_{n=2}^{\infty}\left[(n+1)u_{n}(s,t)a_{n}+\sum_{j=2}^{n-1}(n+1-j)u_{j}(s,t)a_{j}a_{n+-j}\right]z^{n}\right]}$$

$$-\frac{\sum_{n=2}^{\infty}\left[\sum_{j=2}^{n-1}\left(\left[(1-\gamma)+\alpha(\gamma-\beta)\right]u_{n+1-j}(s,t)\right)\right](n+1-j)a_{j}a_{n+1-j}z^{n}}{(1-\beta)(1-\gamma)\left[z+\sum_{n=2}^{\infty}\left[(n+1)a_{n}(s,t)a_{n}+\sum_{j=2}^{n-1}(n+1-j)u_{j}(s,t).u_{n+1-j}(s,t)a_{j}a_{n+-j}\right]z^{n}\right]}$$

$$+\frac{\sum_{n=2}^{\infty}\left[\sum_{j=2}^{n-1}\left(\alpha(1-\beta)(n+1-j)u_{j}(s,t)\right)\right](n+1-j)a_{j}a_{n+1-j}z^{n}}{(1-\beta)(1-\gamma)\left[z+\sum_{n=2}^{\infty}\left[(n+1)a_{n}(s,t)a_{n}+\sum_{j=2}^{n-1}(n+1-j)u_{j}(s,t).u_{n+1-j}(s,t)a_{j}a_{n+-j}\right]z^{n}\right]}\tag{6}$$

$$\leq\frac{\sum_{n=2}^{\infty}\left|(1-\alpha)(1-\gamma)\left(1+u_{n}(s,t)\right)n-\left[(1-\gamma)+\alpha(\gamma-\beta)\right](n+1)u_{n}(s,t)+\alpha(1-\beta)\left(n^{2}+u_{n}(s,t)\right)\right||a_{n}|}{(1-\beta)(1-\gamma)\left[1-\sum_{n=2}^{\infty}(n+1)u_{n}(s,t)|a_{n}|-\sum_{n=2}^{\infty}\left|\sum_{j=2}^{n-1}(n+1-j)u_{j}(s,t).u_{n+1-j}(s,t)a_{j}a_{n+1-j}\right|\right]}$$

$$+\frac{\sum_{n=2}^{\infty}\left|\sum_{j=2}^{n-1}\left(j(1-\alpha)(1-\gamma)u_{n+1-j}(s,t)\right)(n+1-j)a_{j}a_{n+1-j}\right|}{(1-\beta)(1-\gamma)\left[1-\sum_{n=2}^{\infty}(n+1)u_{n}(s,t)|a_{n}|-\sum_{n=2}^{\infty}\left|\sum_{j=2}^{n-1}(n+1-j)u_{j}(s,t).\lambda_{N}(n+1-j)a_{j}a_{n+1-j}\right|\right]}$$

$$-\frac{\sum_{n=2}^{\infty}\left|\sum_{j=2}^{n-1}\left[\left[(1-\gamma)+\alpha(\gamma-\beta)\right]u_{j}(s,t)u_{n+1-j}(s,t)\right](n+1-j)a_{j}a_{n+1-j}\right|}{(1-\beta)(1-\gamma)\left[1-\sum_{n=2}^{\infty}(n+1)u_{n}(s,t)|a_{n}|-\sum_{n=2}^{\infty}\left|\sum_{j=2}^{n-1}(n+1-j)u_{j}(s,t).u_{n+1-j}(s,t)a_{j}a_{n+1-j}\right|\right]}$$

$$+\frac{\sum_{n=2}^{\infty}\left|\sum_{j=2}^{n-1}\left(\alpha(1-\beta)(n+1-j)^{2}u_{j}(s,t)\right)a_{j}a_{n+1-j}\right|}{(1-\beta)(1-\gamma)\left[1-\sum_{n=2}^{\infty}(n+1)u_{n}(s,t)|a_{n}|-\sum_{n=2}^{\infty}\left|\sum_{j=2}^{n-1}(n+1-j)u_{j}(s,t).u_{n+1-j}(s,t)a_{j}a_{n+1-j}\right|\right]}$$

Since

$$k\left|J\left(\alpha,\beta,\gamma,s,t,f(z)\right)-1\right|-\Re\left\{J\left(\alpha,\beta,\gamma,s,t,f(z)\right)-1\right\}\leq(k+1)\left|J\left(\alpha,\beta,\gamma,s,t,f(z)\right)-1\right|,$$

Then

$$\leq \frac{(k+1)\sum_{n=2}^{\infty}\left|(1-\alpha)(1-\gamma)(1+u_n(s,t))n-\left[(1-\gamma)+\alpha(\gamma-\beta)\right](n+1)u_n(s,t)+\alpha(1-\beta)\left(n^2+u_n(s,t)\right)\right||a_n|}{(1-\beta)(1-\gamma)\left[1-\sum_{n=2}^{\infty}(n+1)u_n(s,t)|a_n|-\sum_{n=2}^{\infty}\left|\sum_{j=2}^{n-1}(n+1-j)u_j(s,t).u_{n+1-j}(s,t)a_ja_{n+1-j}\right|\right]}$$

$$+\frac{(k+1)\sum_{n=2}^{\infty}\left|\sum_{j=2}^{n-1}\left(j(1-\alpha)(1-\gamma)-u_j(s,t)\left[(1-\gamma)+\alpha(\gamma-\beta)\right]\right)(n+1-j)u_{n+1-j}(s,t)a_ja_{n+1-j}\right|}{(1-\beta)(1-\gamma)\left[1-\sum_{n=2}^{\infty}(n+1)u_n(s,t)|a_n|-\sum_{n=2}^{\infty}\left|\sum_{j=2}^{n-1}(n+1-j)u_j(s,t).u_{n+1-j}(s,t)a_ja_{n+1-j}\right|\right]}$$

$$+\frac{(k+1)\sum_{n=2}^{\infty}\left|\sum_{j=2}^{n-1}\left(\alpha(1-\beta)(n+1-j)^2u_j(s,t)a_ja_{n+1-j}\right)\right|}{(1-\beta)(1-\gamma)\left[1-\sum_{n=2}^{\infty}(n+1)u_n(s,t)|a_n|-\sum_{n=2}^{\infty}\left|\sum_{j=2}^{n-1}(n+1-j)u_j(s,t).u_{n+1-j}(s,t)a_ja_{n+1-j}\right|\right]}$$

The last expression is bounded by 1 if

$$\sum_{n=2}^{\infty}(k+1)\left|(1-\alpha)(1-\gamma)(1+u_n(s,t))n-\left[(1-\gamma)+\alpha(\gamma-\beta)\right](n+1-j)u_n(s,t)+\alpha(1-\beta)\left(n^2+u_n(s,t)\right)\right||a_n|$$

$$+\sum_{n=2}^{\infty}\left\{\sum_{j=2}^{n-1}(k+1)\left|\left(j(1-\alpha)(1-\gamma)-u_j(s,t)\left[(1-\gamma)+\alpha(\gamma-\beta)\right]\right)(n+1-j)u_{n+1-j}(s,t)a_ja_{n+1-j}\right|\right\}$$

$$+\sum_{n=2}^{\infty}\left\{\sum_{j=2}^{n-1}(k+1)\left|\left(\alpha(1-\beta)(n+1-j)^2u_j(s,t)\right)a_ja_{n+1-j}\right|+(1-\beta)(1-\gamma)(n+1)u_n(s,t)|a_n|\right\}$$

$$+\sum_{n=2}^{\infty}\left\{(1-\beta)(1-\gamma)\sum_{j=2}^{n-1}(n+1-j)u_j(s,t).u_{n+1-j}(s,t)\left|a_ja_{n+1-j}\right|\right\}<(1-\beta)(1-\gamma)$$

This completes the proof.

When $s=1, t=o,$ we have the following result, proved by Khalida Inayat Noor and Sarfraz Nawaz Malik in [6].

Corollary 2.2. A function $f\in A$ and from (1) in the class $k-(\alpha,\beta,\gamma),$ for $-1\leq\beta,\gamma<1,\alpha\geq 0,k\geq 0$ if it satisfies the condition

$$\sum_{n=2}^{\infty}\Psi_n(k;\alpha,\beta,\gamma)<(1-\beta)(1-\gamma), \qquad (7)$$

where

$$\Psi_n(k;\alpha,\beta,\gamma)=(k+1)\left\{(n-1)(10\alpha)(1-\gamma)+n\alpha(1-\beta)(n-1)\right\}|a_n|$$

$$+(k+1)\sum_{j=2}^{\infty}\left\{(j-1)(1-\alpha)(1-\gamma)+\alpha(1-\beta)(n-j)\right\}(n+1-j)\left|a_ja_{n+1-j}\right|$$

$$+(1-\beta)(1-gg)(n+1)|a_n|+(1-\beta)(1-\gamma)\sum_{j=2}^{n-1}(n+1-j)\left|a_ja_{n+1-j}\right|.$$

For $s=1, t=0, \alpha=0,$ we have following result due to Shams and Kulkarni [4].

Corollary 2.3. A function $f\in A$ and from (1) in the class $SD(k,\beta),$ if it satisfies the

$$(1-\beta)(1-\gamma)>\sum_{n=2}^{\infty}\left\{(k+1)(n-1)(1-\gamma)|a_n|+(k+1)\sum_{j=2}^{n-1}(j-1)(1-\gamma)(n+1-j)\left|a_ja_{n+1-j}\right.\right\}$$

condition $+\sum_{n=2}^{\infty}\left\{(1-\beta)(1-\gamma)|a_n|+(1-\beta)(1-\gamma)\sum_{j=2}^{n-1}(n+1-j)\left|a_ja_{n+1-j}\right.\right\}$

$$>(1-\gamma)\sum_{n=2}^{\infty}\left\{(k+1)(n-1)+(1-\beta)\right\}|a_n|.$$

This implies that

$$\sum_{n=2}^{\infty}\{n(k+1)-(k+\beta)\}|a_n|<1-\beta$$

For $s=1,1=0,\alpha=1$ we arrive at Shams and Kulkarni et result in [4].

Corollary 2.4. A function $f\in A$ and from (1) in the class $KD(k,\gamma)$, if it satisfies the condition

$$(1-\beta)(1-\gamma)>\sum_{n=2}^{\infty}\left\{n(k+1)(n-1)(1-\beta)|a_n|+(k+1)\sum_{j=2}^{n-1}(n-j)(n+1-j)(1-\beta)|a_ja_{n+1-j}\right\}$$

$$+\sum_{n=2}^{\infty}\left\{n(1-\beta)(1-\gamma)|a_n|+(1-\beta)(1-\gamma)\sum_{j=2}^{n-1}(n+1-j)|a_ja_{n+1-j}\right\}$$

$$>(1-\beta)\sum_{n=2}^{\infty}n\{(k+1)(n-1)+(1-\gamma)\}|a_n|.$$

This implies that

$$\sum_{n=2}^{\infty}n\{n(k+1)-(k+\gamma)\}|a_n|<1-\gamma$$

Also for $s=1,t=0,gb=0,\gamma=0$ then we get the well-known Kanas's result [7].

Corollary 2.5. A function $f\in A$ and from (1) in the class $UM(\alpha,k)$, if it satisfies the condition

$$\sum_{n=2}^{\infty}\Psi_n(k;\alpha)<1,$$

where

$$\Psi_n(k;\alpha)=(k+1)(n-1)(1-\alpha+n\alpha)|a_n|+(n+1)|a_n|+\sum_{j=2}^{n-1}(n+1-j)|a_ja_{n+1-j}|$$

$$+(k+1)\sum_{j=2}^{n-1}\{(j-1)(1-\alpha)+\alpha(n-j)\}(n+1-j)|a_ja_{n+1-j}|.$$

For $s=1,t=0,\alpha=0,\beta=0,$ then we get result proved by Kanas and Wisniowska in [3].

Corollary 2.6. A function $f\in A$ and from (1) in the class $k-ST$, if it satisfies the condition

$$\sum_{n=2}^{\infty}\{n+k(n-1)\}|a_n|<1.$$

Also for $s=1,t=0,k=0,\alpha=0,$ then we have the following known result, proved by Silverman in [8]

Corollary 2.7. A function $f\in A$ and from (1) in the class $S^*(\beta)$, if it satisfies the condition

$$\sum_{n=2}^{\infty}(n-\beta)|a_n|<1-\beta.$$

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