

Wind Turbine Design for Uncertainties in Design Parameters

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Abstract— The paper analyses the effects of uncertainties in design parameter for a wind turbine design. This paper introduces a simple model of three blade turbine with a fixed support with certain basic assumptions. A meticulous study was done to conclude that uncertainty in wind speed effects the overall deflection of blade tip of wind turbine and performance more than uncertainties in any other parameters such as density of blade, density of air, Young's modulus etc. To consider the uncertainty in design Px model was considered (namely P90, P75 and P50), where $\pm(100-x)\%$ represents the standard deviation from the nominal value. The effects of uncertainty in various design parameters on radial, in-plane, and out of plane deflection of blade tip was analyzed. Finally the results were compared to evaluate the sensitivity of variation of different design parameters on components of deflection of blade tip. Matlab was used as a tool to solve the differential equations and plot the results.

Index Terms— Matlab, probabilistic method, uncertainties, design parameters

I. INTRODUCTION

The lack of electricity affects over 1.6 billion people worldwide (International Energy Agency, 2004). The alternative is to use non conventional wind energy. Wind turbines were used in Persia as early as 200 BC. The Babylonians used windmills for irrigation as early as 1700 BC. The first known practical wind mill were built in Sistan which had 6-12 sails. It found application in sugarcane industries and grist milling.

James Blyth installed first electricity generating windmill which was a battery charging machine in 1887. First modern horizontal axis wind turbine was established in 1931 by USSR. The first megawatt-class wind turbine was established in Vermont. Daniel Halliday's new style of windmill resulted in rapid growth of windmills in America. The growth of wind turbine is unevenly distributed around the world. By 1999, around more than 3/4th wind capacity was installed in Europe and around 1/5th in North America and remaining in Asia.

Even though the development in wind turbine design is century long, there is still scope for development considering the effects of blade aerofoil design, rotor design, material selection etc. In an attempt to improve the wind turbine power curves, Khalfallah and Koliub (2007) focused entirely on rotor and blade design. The behavior of a cantilever beam built into a rigid body was studied by T. R. Kane, et al., (1987). Venkatanarayanan Ramakrishnan and Brian F.

Feeny, (2011) developed the nonlinear partial differential equation of the in-plane motion of a wind turbine blade subject to gravitational loading. Bertagnolio F, et al., (2010), using a conditional simulation technique for stochastic processes, derived the drag and pitching moment by definition a model based on a spectral representation of the aerodynamic lift force. Stochastic modeling of the dynamics of a wind turbine was performed by Mohsen S. Forghani, Thiago G. Ritt using Matlab and Msc.adams (2013).

Small diameter multi blade rotors will run at lower wind speed but they are subjected to high stresses and are unable to withstand extreme wind conditions. Multiple small rotors weigh less than a single large rotor and are less costly and are easier to produce and are less subjected to fatigue. This paper studies the effect of variation in different design parameters such as wind speed, blade density, modulus of elasticity of blade, cross sectional area of blade etc on performance of wind turbine and calculates the effects these parameters has on various stresses in blade design

II. MATHEMATICAL MODELING:

Because of the force exerted by wind, three types of deflections are identified at the blade tip. As the wind blows, low pressure is created on the downstream side of blade. Because of this pressure difference on both sides of blades, the blade is pulled towards low pressure side and hence turbine starts rotating. The force acting on blade is called lift force and this result in transverse deflection of blade tip and is called as inplane deflection. The expression for inplane deflection is given by: (Nomenclature is given in Table 1) (for equations: ref I)

$$\frac{\partial^2}{\partial x^2} \left(EI \frac{\partial^2 \delta_\theta}{\partial x^2} \right) = \frac{1}{2} \times \rho_a \times v_w^2 \times C_l \times h + \rho_b \times g \times A_c \times \sin \theta \quad (1)$$

The second type of deflection is radial deflection which occurs along longitudinal axis of blade and is because of gravity forces and the centrifugal forces. The expression for radial deflection is given by:

$$\delta_r = \frac{\rho_b g R^2}{2E} \cos \theta - \frac{\rho_b R^3 \theta^2}{3E} \quad (2)$$

The third deflection referred to as out of plane deflection acts in the direction of rotation axis. This is perpendicular to the plane of rotation and is due to the drag forces acting on the blade. The expression for out of plane deflection is given by:

$$\frac{\partial^2}{\partial x^2} \left(EI \frac{\partial^2 \delta_z}{\partial x^2} \right) = \frac{1}{2} \times \rho_a \times v_w^2 \times C_d \times h \quad (3)$$

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To calculate the variation in angular velocity of blade tip with time we consider the following equation : (Bagbanci Hasan, 2011; KÜHN M.J 2001; Perdana Abram, 2008)

$$n \int_0^R x \frac{F_1}{R} dx = J\ddot{\theta} + (C + T_g)\dot{\theta} + K \tag{4}$$

Where F_1 is given by $F_1 = \frac{1}{2} \times \rho_a \times v_w^2 \times C_l \times h \times R$ (i)

and $T_g(\theta) = K_g \theta^2$ (ii)

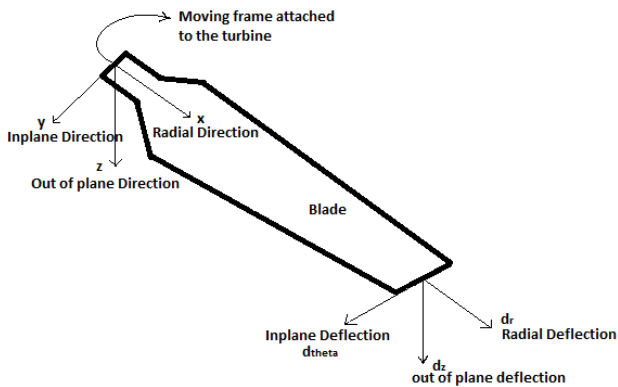


Fig:1 Deflections at blade tip and their directions
The above equations are derived considering the following assumptions [1].

- 1) Material of blade is homogenous isotropic in nature.
- 2) Wind direction is constant and is always perpendicular to the direction of rotation of blade.
- 3) Any other force than centrifugal force, gravitational and aerodynamic force is neglected
- 4) Structural damping of blade is neglected.
- 5) The rotation is about a fixed axis normal to the rotor disk. Then the only moving parts are flexible blades which are rotating about a fixed axis

Following data of a sample wind turbine is considered (Cheney M.C., 1999; Park Joon-Young, et al., 2010) [9]. Table 1. Sample wind turbine design parameters and their nomenclature

Design Parameter	Symbol	Value	Units
Wind speed	v_w	12	m/s
Number of blades	n	3	-
Chord length	h	1.85	m
Air density	ρ_a	1.15	Kg/m ³
Lift coefficient	C_l	1.20	-
Drag coefficient	C_d	0.08	-
Blade length	R	45	m
Moment of inertia of blade	J_b	7.291e6	Kgm ²
Moment of inertia of rotor	J	3* J_b	Kgm ²
Rotational damping of rotor	C	500	Nms/rad
Rotational stiffness of rotor	K	0	Nm/rad

Coef. of rotational resistance	K_g	4.4e5	Nms ² /rad ²
Density of blade material	ρ_b	1600	Kg/m ³
Young modulus of blade material	E	1.45e11	N/m ³
Moment of cross section of blade	I	1	m ⁴
Cross section area of blade	A_c	0.15	m ²

III. P90-P75-P50 MODEL:

It is difficult task to distinguish whether the true potential of wind turbine is not recognized or it is underperforming .The investors needs to know the uncertainty in the performance of wind turbine in order to assess the risks . The energy yield in terms of AEP(annual yield production) is called P50. There is 50% probability that the energy yield will be higher and 50% probability that energy yield will be lower than the predicted value. For P75 and P90 models the risk that annual energy production is not reached is 25% and 10% respectively. Here in this paper performance of wind turbine is studied by considering the uncertainties in different design parameters for P90-P75-P50 model.

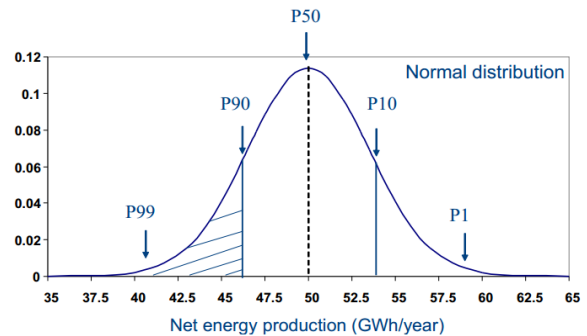


Fig:2. Probabilistic model (source Ref:8)

IV. RESULTS

Matlab was used to solve the differential equations. Solving Eq 4) gives the following graph of angular velocity of wind turbine rotor verses time. From the Fig 3) it can be concluded that it takes about 150 secs for the angular velocity to attain a nearly stable value of 10.7 rad/s.

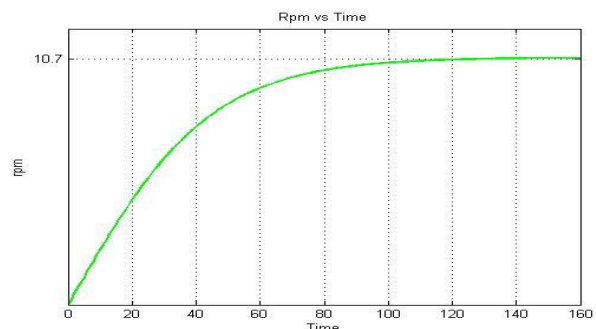


Fig3: Angular velocity (rad/s) of blades vs Time(s) for wind turbine

Now, uncertainties related to the wind speed is considered. Mean value is considered to be equal to nominal value of 12m/s. P90 means that there is 90% probability that wind speed will lie between $12\text{m/s} \pm 10\%$ i.e. $(12 \times 1.1, 12 \times 0.9)$ m/s which is calculated to be (13.2,10.8)m/s. Similarly angular velocity with 75% probability was considered with the standard deviation of $\pm 25\%$. Finally to consider the worst case scenario, P50 model was considered where there is 50% chance that value of wind speed will lie between (12,6) m/s. Upper limit indicates the positive deviation and limit shows negative deviation. With P90 model, the value of angular velocity lies between the (11.9,9.8) rpm. Similarly for P75 and P50 model angular velocity lies between (13.5,8) and (16,5) resp. In Fig4, the large statistical envelop shows that std. deviation increases with time and with increase in uncertainty. Fig 5) shows CV decreases with time which is due to increase in mean velocity. It can be inferred from the figure that with increase in uncertainty (P50), the slope of CV graph becomes steep, this is because of much larger increasing envelope with increasing uncertainty (refer fig4).

Similar analysis is carried out for variation in density of air and variation in value of Cl. (Refer fig 6 and fig 7). Then a comparison is provided for CV of angular velocity for P50 model for the change in wind velocity and density of air(or Cl).(refer fig8) From the graph it can be concluded that the sensitivity of variation in performance is more for change in wind speed than change in density of air (or Cl), this is because of more statistical envelope for change in velocity than change in density(Cl) resulting in a much steeper slope for earlier. Hence from fig 8 it can be concluded that wind velocity plays more imp role than other parameters on angular velocity of blades during starting of wind turbine.

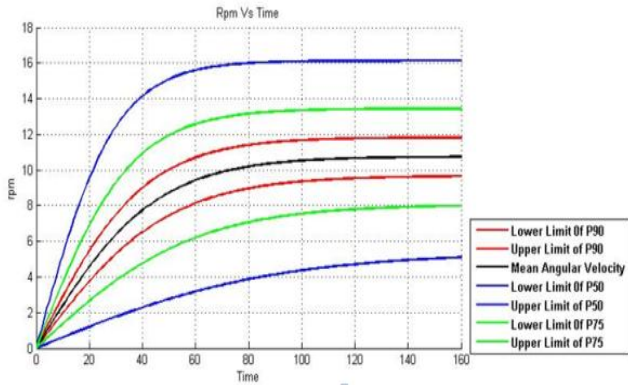


Fig:4 Rpm Vs Time for variation in wind velocity

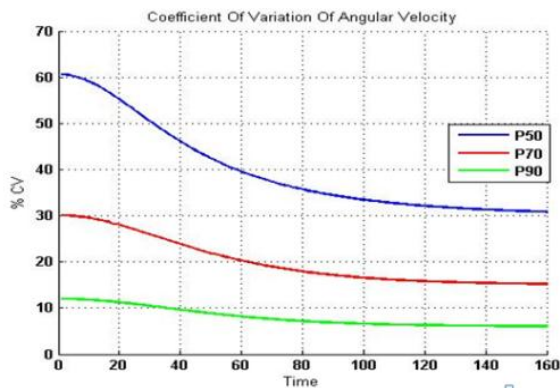


Fig:5. %CV of angular velocity for variation in wind velocity

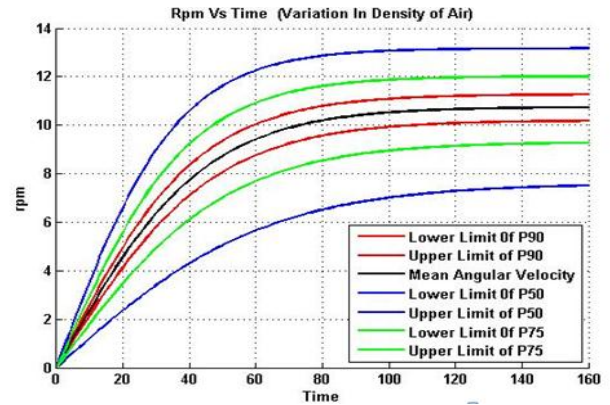


Fig:6 Rpm Vs Time for variation in air density or Cl

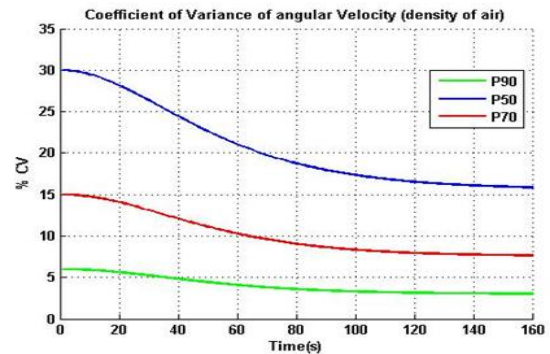


Fig:7 %CV of angular velocity for variation in air density or Cl

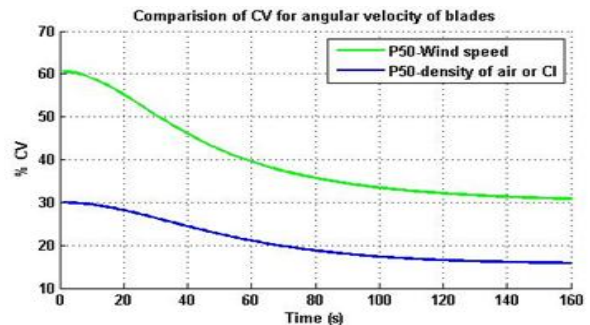


Fig 8: Comparison of CV for angular velocity (P50)

Radial deflection

Radial deflection at initial instance i.e. at $t=0$ is only because of gravity force acting along the longitudinal axis of the blade since at this instance θ is assumed to be 0. Hence for the given wind turbine data the initial deflection due to self weight is 0.1096 mm.

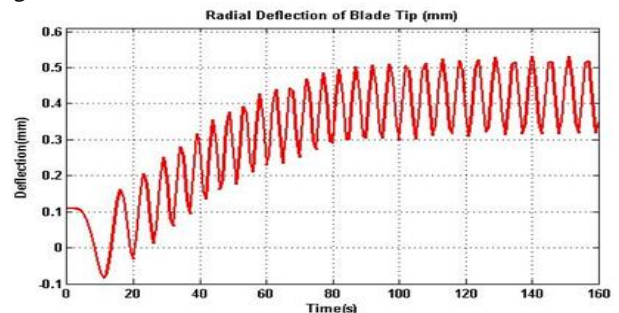


Fig:9 Radial deflection of blade tip

Now the uncertainty in wind velocity V_w , Density of air, Young's modulus, lift coefficient and density of blade is considered. Graphs for CV are shown for all these

iterations for P50, P75 and P90 models. Peaks in the graphs refer to the points where mean radial deflection has zero value. From fig:12 it can be concluded that sensitivity of radial deflection for small variation in wind velocity is large as compared to the large variation in wind velocity which is in contrast with variation in every other parameters. Comparison is provided for CV of radial deflection for various parameters for P50 model in fig:14 from which it can be concluded that sensitivity of radial deflection for variation in wind velocity is more.

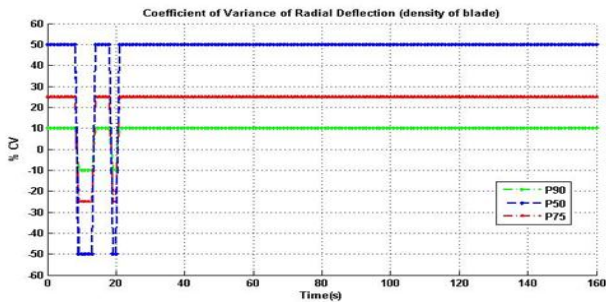


Fig:10. % CV of radial deflection (density of blade)

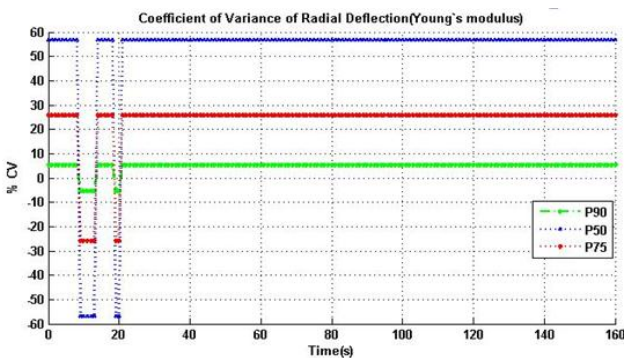


Fig:11. % CV of radial deflection (Young's modulus)

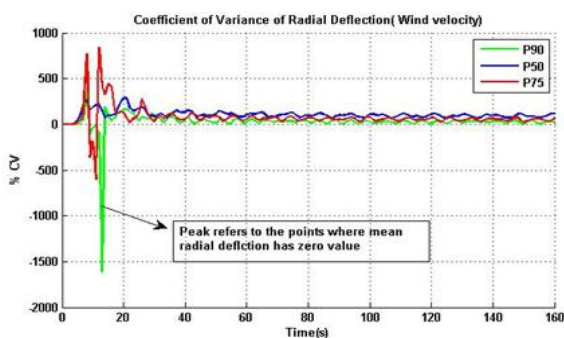


Fig:12. % CV of radial deflection (Wind Velocity)

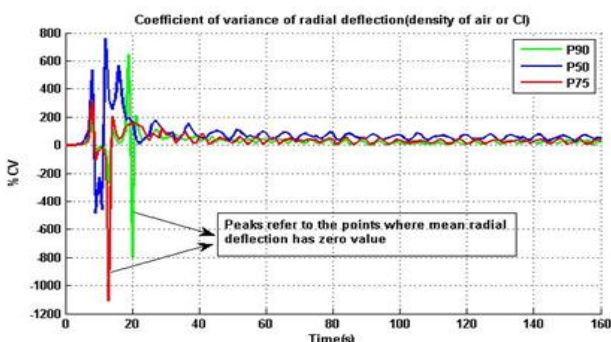


Fig:13 % CV of radial deflection (Air density or Cl)

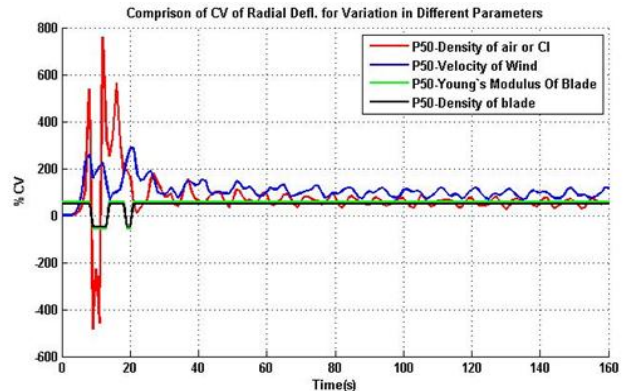


Fig14: Comparison of CV for radial deflection (P50)

Out of plane Deflection

The out of plane deflection is independent of time. For the given wind turbine data we get 0.043mm as value for out of plane deflection (fig16). Now considering the P90, P75 and P50 model out of plane deflection are calculated for variation in wind velocity, air density, Cd, E and I (fig17 –fig20). A comparison is provided for the CV for P50 model (fig 21). Hence out of plane deflection is more sensitive to variation in wind velocity because of greater envelop.

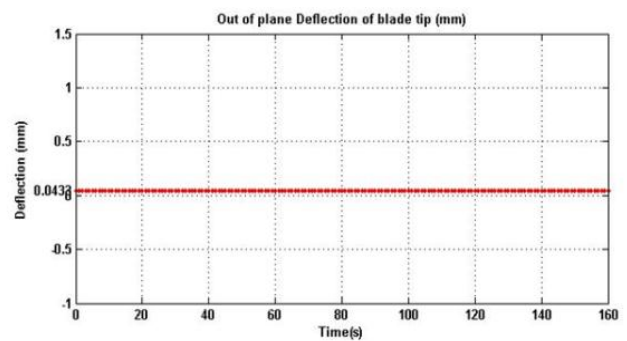


Fig:16: Out of plane deflection of blade Tip

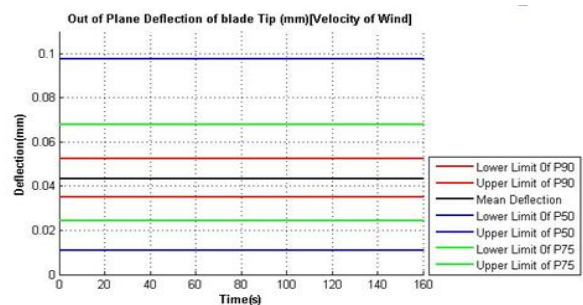


Fig17. Out of plane defl. for variation in wind speed

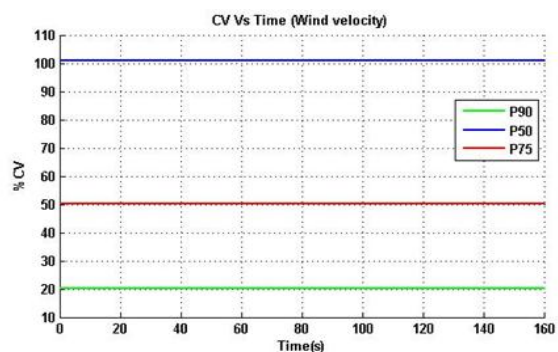


Fig18. %CV of out of plane defl. (wind velocity)

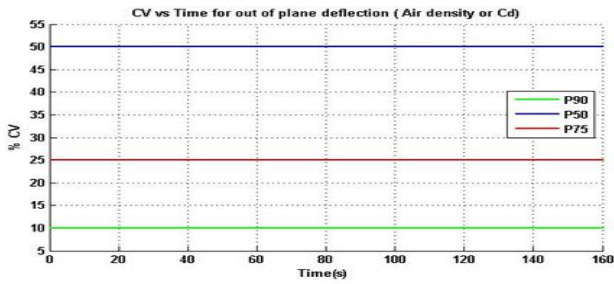


Fig19. % CV of Out of plane defl. (air density or Cd)

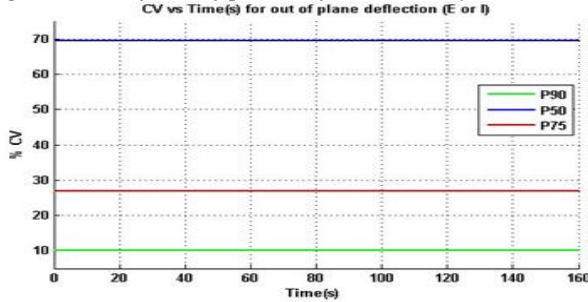


Fig20.%CV of out of plane defl. (E or I)

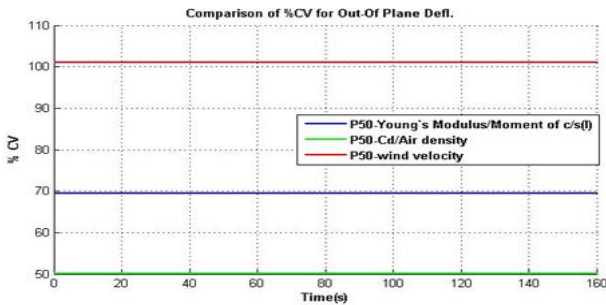


Fig21. Comparison of % CV of out of plane defl.(P50)

Inplane Deflection

We see initial value for inplane deflection at $t=0$ because of the first term on RHS of eq1) where we considered initial force acting on the turbine because of power exerted by the wind. Inplane deflection plays major part in total deflection of blade tip because of higher magnitude of deflection as compared to other deflections (fig 22). A similar analysis is performed for wind velocity, density of blade, E, I, Cl and air density(fig23-fig26). Comparison is provided for P50 model (fig 27). Hence inplane deflection is more sensitive to wind velocity.

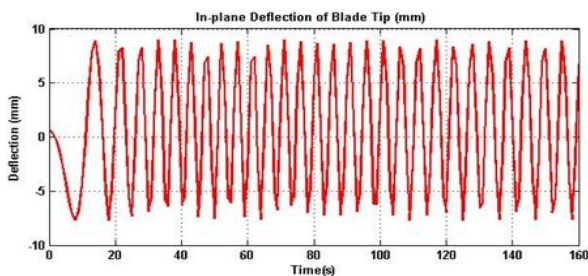


Fig22: In-plane deflection of blade tip

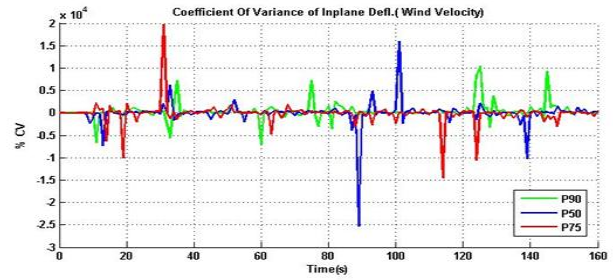


Fig:23 % CV of inplane defl.(deflection) (Wind velocity)

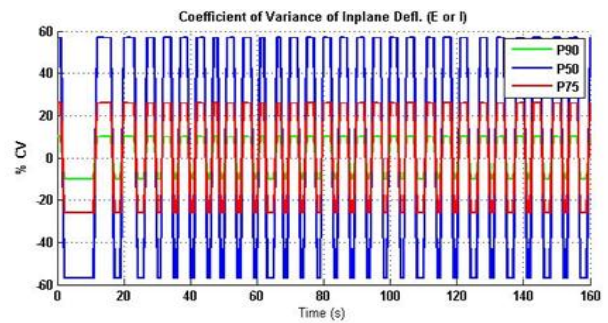


Fig:24.% CV of inplane defl.(E or I).

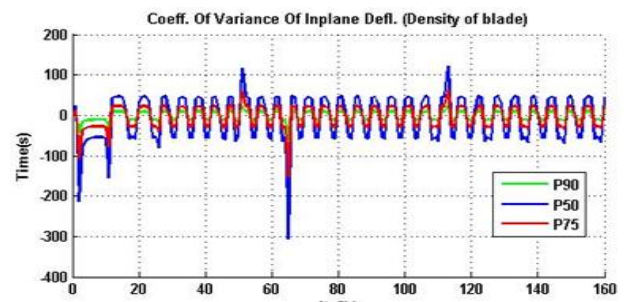


Fig:25. % CV of inplane defl. (Density Of blade)

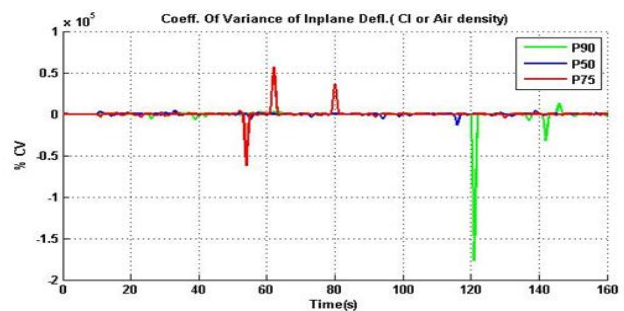


Fig:26.% CV of inplane defl.(Cl or air density)

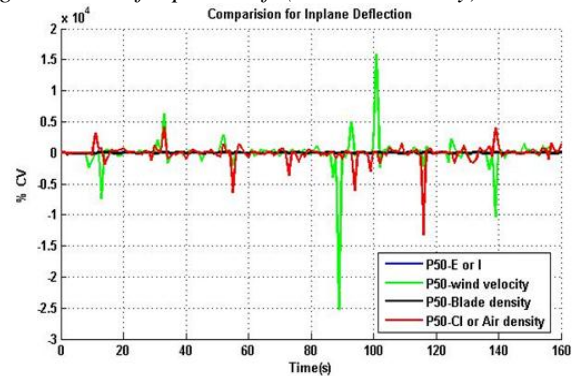


Fig27: Comparison of CV for inplane deflection

1.7 Total deflection :

The total deflection is calculated using equation 5)(fig 28). Overall we can conclude that this wind turbine model is more sensitive to variation in wind speed than any other parameter in both cases of deflection and initial rotation of wind turbine.

$$\delta_T = \sqrt{\delta_z^2 + \delta_\theta^2 + \delta_T^2} \tag{5}$$

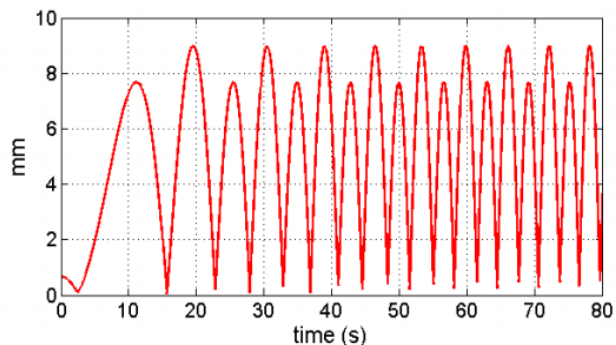


Fig:28. Total Deflection of blade tip.

V. CONCLUSION

A wind turbine is modeled using basic assumptions to calculate radial , inplane , out of plane deflection. A rigorous study was performed to evaluate the effect of variation in various parameters on performance of wind turbine. It was proved that variation in wind speed plays the most significant role in overall deflection and performance as compared to variation in other parameters like density of blade , density of air, Cl ,Cd, Young`s modulus (E) and moment of C/s of blade (I). A probabilistic approach was used to consider the uncertainties. The analysis was carried out by considering P90-P75 and P50 model to consider the worst case scenario. Comparisons were provided for coefficient of variance for each deflection for variation in different parameters to justify the conclusion.

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