A MODEL FOR OPTIMISATION OF COMPRESSIVE STRENGTH OF SAND-LATERITE BLOCKS USING OSADEBE'S REGRESSION THEORY

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Abstract — **Blocks which are essential masonry units used in construction, can be made from a wide variety of materials. In some places, sand is scarce and in extreme cases, it is not available. River sand is a widely used material in block production. Various methods adopted to obtain correct mix proportions to yield the desired strength of the blocks, have limitations. To reduce dependence on river sand, and optimise strength of blocks, this work presents a mathematical model for optimisation of compressive strength of sand-laterite blocks using Osadebe's regression method. The model can predict the mix proportion that will yield the desired strength and vice-versa. Statistical tools were used to test the adequacy of the model and the result is positive.**

***Index Terms*—Compressive strength, Model, Optimisation, Sand-laterite blocks, Regression theory.**

I. INTRODUCTION

Shelter, is one of the basic needs of man. One of the major materials used in providing this shelter is blocks. These blocks are essential materials commonly used as walling units in the construction of shelter. They can be made from a wide variety of materials ranging from binder, water, sand, laterite, coarse aggregates, clay to admixtures. The constituent materials determine the cost of the blocks, which also determines the cost of shelter.

The most common type of block in use today is the sandcrete block which is made of cement, sand and water. Presently, there is an increasing rise in the cost of river sand and this affects the production cost of blocks. Consequently, it has made housing units unaffordable for middle class citizens of Nigeria. In some parts of the country like Northern Nigeria, there is scarcity or even non availability of river sand. This also affects block production in these areas. Sand dredging process has its own disadvantages. There is need for a reduced dependence on this river sand in block production. Hence, an alternative material like laterite, which is readily available and affordable in most part of the country, can be used to achieve this purpose.

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To obtain any desired property of blocks, the correct mix proportion has to be used. Various mix design methods have been developed in order to achieve the desired property of blocks. It is a well known fact that the methods have some limitations. They are not cost effective, and time and energy are spent in order to get the appropriate mix proportions.

Several researchers [1]-[5] have worked on alternative materials in block production. Optimisation of aggregate composition of laterite/sand hollow block using Scheffe's simplex theory has been carried out [6]. Consequently, this paper presents the optimisation of compressive strength of sand- laterite blocks using Osadebe's regression theory.

II. MATERIALS AND METHODS

A. Materials

The materials used in the production of the blocks are cement, river sand, laterite and water. Dangote cement brand of ordinary Portland cement with properties conforming to BS 12 was used [7]. The river sand was obtained from Otamiri River, in Imo State. The laterite was sourced from Ikeduru LGA, Imo State. The grading and properties of these fine aggregates conformed to BS 882 [8]. Potable water was used.

B. Methods

Two methods, namely analytical and experimental methods were used in this work.

Analytical method

Osadebe [9] assumed that the following response function, $F(z)$ is differentiable with respect to its predictors, Z_i.

$$F(z) = F(z^{(0)}) + \sum [\partial F(z^{(0)}) /\partial z_i](z_i - z_i^{(0)}) + ½! \sum\sum [\partial^2 F(z^{(0)})/\partial z_i \partial z_j](z_i - z_i^{(0)})(z_j - z_j^{(0)}) + ½! \sum\sum [\partial^2 F(z^{(0)}) / \partial z_i^2] (z_i - z_i^{(0)})^2 + \ldots\ldots$$

where $1 \le i \le 4$, $1 \le i \le 4$, $1 \le j \le 4$,and $1 \le i \le 4$ respectively.

z_i = fractional portions or predictors

= ratio of the actual portions to the quantity of mixture

By making use of Taylor's series, the response function could be expanded in the neighbourhood of a chosen point:

$$Z^{(0)} = Z_1^{(0)}, Z_2^{(0)}, Z_3^{(0)}, Z_4^{(0)}, Z_5^{(0)} \quad (2)$$

This function was used to derive the following optimisation model equation. Its derivation is contained in the reference [10].

$$Y = \alpha_1 z_1 + \alpha_2 z_2 + \alpha_3 z_3 + \alpha_4 z_4 + \alpha_{12} z_1 z_2 + \alpha_{13} z_1 z_3 + \alpha_{14} z_1 z_4 + \alpha_{23} z_2 z_3 + \alpha_{24} z_2 z_4 + \alpha_{34} z_3 z_4 \quad (3)$$

In general, Eqn (3) is given as:

$$Y = \sum \alpha_i z_i + \sum \alpha_{ij} z_i z_j \quad (4)$$

where $1 \leq i \leq j \leq 4$

Eqns (3) and (4) are the optimization model equations and Y is the response function at any point of observation, z_i are the predictors and α_i are the coefficients of the optimization model equations.

Determination of the coefficients of the optimisation model

Different points of observation have different responses with different predictors at constant coefficients. At the nth observation point, $Y^{(n)}$ will correspond with $Z_i^{(n)}$. That is to say that:

$$Y^{(n)} = \sum \alpha_i z_i^{(n)} + \sum \alpha_{ij} z_i^{(n)} z_j^{(n)} \quad (5)$$

where $1 \leq i \leq j \leq 4$ and n = 1,2,3, …………. 10

Eqn (5) can be put in matrix from as

$$[Y^{(n)}] = [Z^{(n)}] \{\alpha\} \quad (6)$$

Rearranging Eqn (6) gives:

$$\{\alpha\} = [Z^{(n)}]^{-1} [Y^{(n)}] \quad (7)$$

The actual mix proportions, $s_i^{(n)}$ and the corresponding fractional portions, $z_i^{(n)}$ are presented on Table 1. These values of the fractional portions $Z^{(n)}$ were used to develop $Z^{(n)}$ matrix (see Table 2) and the inverse of $Z^{(n)}$ matrix. The values of $Y^{(n)}$ matrix will be determined from laboratory tests. With the values of the matrices $Y^{(n)}$ and $Z^{(n)}$ known, it is easy to determine the values of the constant coefficients α_i of Eqn (7).

Experimental method

The actual mix proportions were measured by weight and used to produce sand-laterite solid blocks of size 450mm x 150mm x 225mm. The blocks were demoulded immediately after manual the compaction of the newly mixed constituent materials in a mould. The blocks were cured for 28 days after 24 hours of demoulding using the environmental friendly method of covering with tarpaulin/water proof devices to prevent moisture loss. In accordance to BS 2028 [11], the blocks were tested for compressive strength. Using the universal compression testing machine, blocks were crushed and the crushing load was recorded and used to compute the compressive strength of blocks.

Table 1: Values of actual mix proportions and their corresponding fractional portions for a 4-component mixture

N	S_1	S_2	S_3	S_4	RESPONSE	Z_1	Z_2	Z_3	Z_4
1	0.8	1	3.2	4.8	Y_1	0.08163	0.10204	0.32653	0.4898
2	1	1	3.75	8.75	Y_2	0.06897	0.06897	0.25862	0.60345
3	1.28	1	3.334	13.336	Y_3	0.06755	0.05277	0.17594	0.70375
4	2.2	1	2.5	22.5	Y_4	0.07801	0.03546	0.08865	0.79787
5	0.9	1	3.475	6.775	Y_{12}	0.07407	0.0823	0.28601	0.55761
6	1.04	1	3.267	9.068	Y_{13}	0.07235	0.06957	0.22727	0.63082
7	1.5	1	2.85	13.65	Y_{14}	0.07895	0.05263	0.15	0.71842
8	1.14	1	3.542	11.043	Y_{23}	0.06816	0.05979	0.21178	0.66027
9	1.6	1	3.125	15.625	Y_{24}	0.07494	0.04684	0.14637	0.73185
10	1.74	1	2.917	17.918	Y_{34}	0.07381	0.04242	0.12373	0.76004

Table 2: $Z^{(n)}$ matrix for a 4-component mixture

Z_1	Z_2	Z_3	Z_4	Z_1Z_2	Z_1Z_3	Z_1Z_4	Z_2Z_3	Z_2Z_4	Z_3Z_4
0.08163	0.10204	0.32653	0.4898	0.00833	0.02666	0.03998	0.03332	0.04998	0.15993
0.06897	0.06897	0.25862	0.60345	0.00476	0.01784	0.04162	0.01784	0.04162	0.15606
0.06755	0.05277	0.17594	0.70375	0.00356	0.01188	0.04754	0.00928	0.03714	0.12381
0.07801	0.03546	0.08865	0.79787	0.00277	0.00692	0.06225	0.00314	0.02829	0.07073
0.07407	0.0823	0.28601	0.55761	0.0061	0.02119	0.0413	0.02354	0.04589	0.15948
0.07235	0.06957	0.22727	0.63082	0.00503	0.01644	0.04564	0.01581	0.04388	0.14337
0.07895	0.05263	0.15	0.71842	0.00416	0.01184	0.05672	0.00789	0.03781	0.10776
0.06816	0.05979	0.21178	0.66027	0.00408	0.01444	0.045	0.01266	0.03948	0.13983
0.07494	0.04684	0.14637	0.73185	0.00351	0.01097	0.05485	0.00686	0.03428	0.10712
0.07381	0.04242	0.12373	0.76004	0.00313	0.00913	0.0561	0.00525	0.03224	0.09404

III. RESULTS AND ANALYSIS

Three blocks were tested for each point and the average taken as the compressive strength at the point.

The test results of the compressive strength of the sand-laterite blocks based on 28-day strength are presented as part of Table 3. The compressive strength was obtained from the following equation:

$$f_c = P/A \quad (8)$$

where f_c = the compressive strength

P = crushing load

A = cross-sectional area of the specimen

Table 3: Compressive strength test result and replication variance

Expt. No.	Replicates	Response Y_i (N/mm^2)	Response Symbol	$\sum Y_i$	Y	$\sum Y_i^2$	S_i^2
1	1A 1B 1C	2.667 3.259 3.111	Y_1	9.037	3.012	27.412	0.095
2	2A 2B 2C	1.926 2.074 2.074	Y_2	6.074	2.025	12.312	0.007
3	3A 3B 3C	1.482 1.630 1.778	Y_3	4.890	1.630	8.015	0.022
4	4A 4B 4C	1.185 1.333 1.259	Y_4	3.777	1.259	4.766	0.005
5	5A 5B 5C	2.519 2.074 2.370	Y_{12}	6.963	2.321	16.264	0.051
6	6A 6B 6C	1.482 2.667 2.074	Y_{13}	6.223	2.074	13.610	0.234
7	7A 7B 7C	1.630 1.704 1.778	Y_{14}	5.112	1.704	8.722	0.004
8	8A 8B 8C	2.074 1.778 1.926	Y_{23}	5.778	1.926	11.172	0.022
9	9A 9B 9C	0.889 1.333 1.333	Y_{24}	3.555	1.185	4.344	0.066
10	10A 10B 10C	0.889 1.333 1.482	Y_{34}	3.704	1.235	4.764	0.095
			Control				
11	11A 11B 11C	1.778 2.222 2.074	C_1	6.073	2.024	12.400	0.053
12	12A 12B 12C	2.074 1.926 1.926	C_2	5.926	1.975	11.720	0.007
13	13A 13B 13C	2.519 3.259 2.222	C_3	8.000	2.666	21.904	0.285
14	14A 14B 14C	2.074 1.777 1.926	C_4	5.777	1.926	11.161	0.022
15	15A 15B 15C	2.074 1.778 2.074	C_5	5.926	1.975	11.764	0.029
16	16A 16B 16C	2.074 1.778 1.778	C_6	5.630	1.876	10.624	0.029
17	17A 17B 17C	2.222 2.074 2.222	C_7	6.518	2.173	14.176	0.007
18	18A 18B 18C	1.630 1.452 1.630	C_8	4.712	1.571	7.422	0.011
19	19A 19B 19C	1.185 1.185 1.259	C_9	3.629	1.210	4.394	0.002
20	20A 20B 20C	1.185 1.185 1.037	C_{10}	3.407	1.136	3.884	0.007
						$\sum$	1.053

The values of the mean of responses, Y and the variances of replicates S_i^2 presented in columns 5 and 8 of Table 3, are gotten from the following Eqns (9) and (10):

$$Y = \sum_{i=1}^{n} Y_i/n \qquad (9)$$

$$S^2_i = [1/(n-1)]\{\sum Y_i^2 - [1/n(\sum Y_i)^2]\} \qquad (10)$$

where 1≤i≤n and this equation is an expanded form of Eqn (9)

$$S^2_i = [1/(n-1)][\sum_{i=1}^{n} (Y_i - Y)^2] \qquad (11)$$

where Y_i = responses
Y = mean of the responses for each control point
n = number of parallel observations at every point
n−1 = degree of freedom
S^2_i = variance at each design point
Considering all the design points, the number of degrees of freedom, V_e is given as

$$V_e = \sum N - 1 = 20 - 1 = 19 \qquad (12)$$

where N is the number of design points
Replication variance can be found as follows:

$$S^2_y = (1/V_e) \sum_{i=1}^{N} S_i^2 = 1.053/19 = 0.055 \qquad (13)$$

where S_i^2 is the variance at each point
Using Eqns (12) and (13), the replication error, S_y can be determined as follows:

$$S_y = \sqrt{S^2_y} = \sqrt{0.05} = 0.235 \qquad (14)$$

This replication error value was used below to determine the t-statistics values for the model

A. Determination of the final optimisation model for compressive strength of sand-laterite blocks

Substituting the values of $Y^{(n)}$ from the test results (given in Table 3) into Eqn (7), gives the values of the coefficients, α as:
$α_1$=-6966.045, $α_2$ = -14802.675, $α_3$ = -418.035, $α_4$ = -27.196, $α_5$ = 47847.731, $α_6$ = 1380.941, $α_7$ = 7862.325, $α_8$ = 20697.830, $α_9$ = 13162.925, $α_{10}$ = 842.339
Substituting the values of these coefficients, α into Eqn (3) yields:

$$Y = -6966.045Z_1 - 14802.675Z_2 - 418.035Z_3 - 27.196Z_4 + 47847.731Z_5 + 1380.941Z_6 + 7862.325Z_7 + 20697.830Z_8 + 13162.925Z_9 + 842.339Z_{10} \qquad (15)$$

Eqn (15) is the model for optimisation of compressive strength of sand- laterite block based on 28-day strength.

B. Test of adequacy of optimisation model for compressive strength of sand-laterite blocks (11)

The model was tested for adequacy against the controlled experimental results. The hypotheses for this model are as follows:
Null Hypothesis (H_o): There is no significant difference between the experimental and the theoretically estimated results at an α- level of 0.05Alternative Hypothesis (H_1): There is a significant difference between the experimental and theoretically expected results at an α –level of 0.05.

The student's t-test and fisher test statistics were used for this test. The expected values ($Y_{predicted}$) for the test control points, were obtained by substituting the values of Z_i from Z^n matrix into the model equation i.e. Eqn (15). These values were compared with the experimental result ($Y_{observed}$) from Table 3. (13)
Student's test
For this test, the parameters $Δ_Y$, Є and t are evaluated using the following equations respectively

$$Δ_Y = Y_{(observed)} - Y_{(predicted)} \qquad (16)$$

$$Є = (\sum a_i^2 + \sum a_{ij}^2) \qquad (17)$$

$$t = Δy\sqrt{n} / (Sy\sqrt{1+ Є}) \qquad (18)$$

where Є is the estimated standard deviation or error,
t is the t-statistics,
n is the number of parallel observations at every point
S_y is the replication error
a_i and a_{ij} are coefficients while i and j are pure components
$a_i = X_i(2X_i-1)$
$a_{ij} = 4X_iX_j$
$Y_{obs} = Y_{(observed)}$ = Experimental results
$Y_{pre} = Y_{(predicted)}$ = Predicted results
The details of the t-test computations are given in Table 4.

C. T-value from standard statistical table

For a significant level, α = 0.05, $t_{α/l}(v_e) = t_{0.05/10}(9) = t_{0.005}(9)$ = 3.250. The t-value is obtained from standard t-statistics table.
This value is greater than any of the t-values obtained by calculation (as shown in Table 4). Therefore, we accept the Null hypothesis. Hence the model is adequate.

Table 4: T-statistics test computations for Osadebe's compressive strength model

N	CN	i	j	a_i	a_{ij}	a_i^2	a_{ij}^2	$ε$	$y_{(observed)}$	$y_{(predicted)}$	$Δ_Y$	t
1	C_1	1	2	-0.125	0.25	0.0156	0.0625					
		1	3	-0.125	0.25	0.0156	0.25					
		1	4	-0.125	0	0.0156	0					
		2	3	-0.125	0.5	0.0156	0.25					
		2	4	-0.125	0	0.0156	0					
		3	4	0	0	0	0					
		4	-	0	-	0	0					
					∑	0.0781	0.5625	0.6406	2.024	1.985	0.039	0.224
Similarly												
2		-	-	-	-	-	-	0.625	1.975	2.104	0.129	0.223
3		-	-	-	-	-	-	0.963	2.666	2.493	0.173	0.910
4		-	-	-	-	-	-	0.899	1.926	1.991	0.065	0.348
5		-	-	-	-	-	-	0.669	1.975	2.058	0.083	0.474

6		-	-	-	-	-	-	0.650	1.876	1.971	0.095	0.545
7		-	-	-	-	-	-	0.609	2.173	2.239	0.066	0.383
8		-	-	-	-	-	-	0.484	1.571	1.568	0.003	0.018
9		-	-	-	-	-	-	0.609	1.210	1.232	0.022	0.128
10		-	-	-	-	-	-	0.734	1.136	1.185	0.049	0.274

D. Fisher Test

For this test, the parameter y, is evaluated using the following equation:

$y = \sum Y/n$ (19)

where Y is the response and n the number of responses.

Using variance, $S^2 = [1/(n-1)][\sum (Y-y)^2]$ and $y = \sum Y/n$ for $1 \leq i \leq n$ (20)

The computation of the fisher test statistics is presented in Table 5.

(19)

Table 5: F-statistics test computations for Osadebe's compressive strength model

Response Symbol	$Y_{(observed)}$	$Y_{(predicted)}$	$Y_{(obs)} - y_{(obs)}$	$Y_{(pre)} - y_{(pre)}$	$(Y_{(obs)} - y_{(obs)})^2$	$(Y_{(pre)} - y_{(pre)})^2$
C_1	2.024	1.985	0.1708	0.1024	0.029173	0.010486
C_2	1.975	2.104	0.1218	0.2214	0.014835	0.049018
C_3	2.666	2.493	0.8128	0.6104	0.660644	0.372588
C_4	1.926	1.991	0.0728	0.1084	0.0053	0.011751
C_5	1.975	2.058	0.1218	0.1754	0.014835	0.030765
C_6	1.876	1.971	0.0228	0.0884	0.00052	0.007815
C_7	2.173	2.239	0.3198	0.3564	0.102272	0.127021
C_8	1.571	1.568	-0.2822	-0.3146	0.079637	0.098973
C_9	1.21	1.232	-0.6432	-0.6506	0.413706	0.42328
C_{10}	1.136	1.185	-0.7172	-0.6976	0.514376	0.486646
$\sum$	18.532	18.826			1.835298	1.618342
	$y_{(obs)}$=1.8532	$Y_{(pre)}$=1.8826				

Legend: $y = \sum Y/n$

where Y is the response and n the number of responses.

Using Eqn (20), $S^2_{(obs)}$ and $S^2_{(pre)}$ are calculated as follows:

$S^2_{(obs)} = 1.835298/9 = 0.2039$ and $S^2_{(pre)} = 1.618342/9 = 0.1798$

The fisher test statistics is given by:

$F = S_1^2 / S_2^2$ (21)

where S_1^2 is the larger of the two variances.

Hence, $S_1^2 = 0.2039$ and $S_2^2 = 0.1798$

Therefore, $F = 0.2039/0.1798 = 1.134$

From standard Fisher table, $F_{0.95}(9,9) = 3.25$, which is higher than the calculated F-value. Hence the regression equation is adequate.

IV. CONCLUSION

From this study, the following conclusions can be made:

1. Osadebe's regression theory has been applied and used successfully to develop model for optimisation of compressive strength of sand-laterite blocks.
2. The student's t-test and the fisher test used in the statistical hypothesis showed that the model developed is adequate.
3. The optimisation model can predict values of compressive strength of sand-laterite blocks if given the mix proportions and vice versa.

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