Legendre, Grimm, Balanced Prime, Prime triplet, Polignac's conjecture, a problem and 17 tips with proof to solve problems on number theory

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Abstract— **In this paper we will prove five conjectures, one problem viz.**

1) **Legendre Conjecture.**
2) **Grimm's Conjecture.**
3) **Balanced Primes are infinite.**
4) **Prime triplets are infinite.**
5) **Polignac's conjecture.**
6) **Can a prime p satisfy $2^{p-1} \equiv 1 \pmod{p^2}$ and $3^{p-1} \equiv 1 \pmod{p^2}$ simultaneously?**

And 17 tips with proof to solve problems on number theory.

Index Terms— **Legendre Conjecture, Balanced Primes, Polignac's conjecture.**

I. Introduction

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B. What is prime

A number which is divisible by 1 and that number only is called a prime number.

I suggest another definition of prime is : When we adorn odd numbers in increasing series in natural number line starting with 1,2,3 there is gap of odd numbers. This gap numbers are filled with prime numbers. See the example below under "How primes generate"

All odd composite number can be written in the form : ΠΠΠ...Π..(3^i*5^j*7^k......all prime numbers where i,j,k,..... runs from 0 to infinity.

C. How primes generate

Prime numbers are odd numbers (except 2). Let's say 1,2,3 only these 3 numbers are given and we are supposed to make natural numbers.

1 2 3 2^2 2*3 but 2^2 and 2*3 both even number. But between two even numbers there must be an odd number . We call it 5 a prime number because no other integer can give birth to this number. Prime number series generates in this way.

II. Legendre's Conjecture

Between 132 (=169) and 142 (=196) there are five primes (173, 179, 181, 191, and 193); between 302 (=900) and 312 (=961) there are eight primes (907, 911, 919, 929, 937, 941, 947, and 953); between 352 (=1225) and 362 (=1296) there are ten primes (1229, 1231, 1237, 1249, 1259, 1277, 1279, 1283, 1289, and 1291).

The problem is to prove Legendre's Conjecture, which states that there is at least one prime number between every pair of consecutive squares, or find a counter-example.

Solution

Let's say n is even and (n+1) is odd.

Let's take $n^2 + 1 = 3m$ (i.e. $n^2 + 1$ is divisible by 3 and $(n+1)^2 - 2 = 3(m+r)$

Now, between 2 consecutive odd multiple of 3 there are 2 odd numbers. (e.g. between 9 and 15 there 2 odd numbers 11 and 13)

So, there are r number of gaps between 3m and 3(m+r)

So, in the gaps there are 2r odd numbers and (r+1) numbers are divisible by 3.

⇨ There are total 2r + (r+1) = 3r +1 number of odd numbers between n^2 and $(n+1)^2$

Now, $(n+1)^2 - n^2 = 2n+1$

⇨ There are (2n+1)-1 =2n numbers between n^2 and $(n+1)^2$

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⇨ There are n odd numbers between n^2 and $(n+1)^2$

Therefore, n = 3r + 1
So, there is no multiple of numbers after (m+r) in between n^2 and $(n+1)^2$ because 3 is least odd number.
Now, there are (m+r)/2 – 1 number of odd numbers before (m+r)
Now, after 3 next odd number is 5.
Now, if s is multiplied by 3 gives the number 3(m+r) and (s-2) multiplied by 5 gives the previous odd number of 3(m+r) and (s-4) multiplied by 7 gives 3(m+r) – 4 and so on.... then there are {5(s-2) – 3s} numbers do not produce any multiple between n^2 and $(n+1)^2$ because after 3 next odd number is 5. Similarly, there are {7(s-4 – 5(s-2)} number of numbers do not produce any multiple between n^2 and $(n+1)^2$ because after 5 next odd number s 7. And so on. This is worst possible scenario because if we take more difference then number of numbers which doesn't produce any odd multiple between n^2 and $(n+1)^2$ will be more. But we are considering less.
Now we will calculate the number of numbers which do not produce multiple between n^2 and $(n+1)^2$ and less than (m+r).
The series is as foloows :
{5(s-2) – 3(s-0)} = 2s – 5*2 + 3*0
{7(s-4) – 5(s-2)} = 2s – 7*4 + 5*2
{9(s-6) – 7(s-4)} = 2s – 9*6 + 7*4
...
..
..
[(2n+3)(s-2n) – (2n+1){s-(2n-2)}] = 2s – (2n+3)*2n + (2n+1)(2n-2)

Adding we get, 2ns – 2n(2n+3) (s = m+r here)
So, this is the number of numbers which doesn't produce any multiple between n^2 and $(n+1)^2$.
Now, when we multiply a number the quotient also a multiple of that number at the same time except the square numbers. Because square numbers have odd number of multiples. But between two consecutive square number there cannot be any square number. So, we are fine to take number of odd multiples before (m+r) as (m+r)/2 – 1.
So, number of numbers that produce multiple between n^2 and $(n+1)^2$ is :
(m+r) -1 - {2n(m+r) – 2n(2n+3)}
= (m+r) – 1 – 2n(m+r) + 2n(2n+3)
Now, there are n numbers between n^2 and $(n+1)^2$
So, if we prove n – [(m+r) – 1 – 2n(m+r) + 2n(2n+3)] is positive then it means there are odd numbers between n^2 and $(n+1)^2$ which is not born by multiplying any number. (Implying they are prime according to alternate definition).
Now putting m = $(n^2+1)/3$ and r = (n-1)3 from above and simplifying we get,
$(4n^3 - 29n^2 - 17n + 3)/12$
Which is greater than 0 for n >7.
Upto n = 7 we can check manually that there is at least one prime between n^2 and $(n+1)^2$
Between 2^2 and 1^2 there is 3
Between 3^2 and 2^2 there is 5, 7
Between 4^2 and 3^2 there is 11, 13
Between 5^2 and 4^2 there is 17, 19, 23
Between 6^2 and 5^2 there is 29, 31
Between 7^2 and 6^2 there is 37, 41, 43, 47.

Result

1) So, we have seen manually upto n = 7 that there is at least one prime between n^2 and $(n+1)^2$
2) We have proved that there is at least one prime between n^2 and $(n+1)^2$ for n > 7.

Conclusion

Legendre's Conjecture is true.

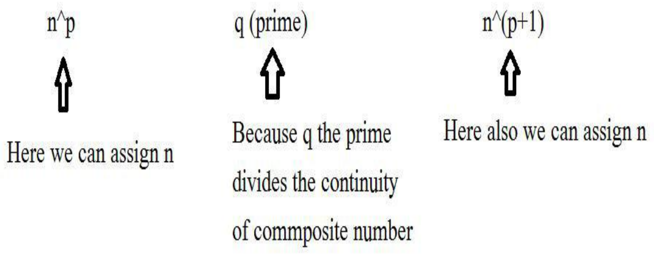
III. Grimm's Conjecture

Grimm's conjecture states that to each element of a set of consecutive composite numbers one
can assign a distinct prime that divides it.
For example, for the range 242 to 250, one can assign distinct primes as follows:
242: 11 243: 3 244: 61 245: 7 246: 41 247: 13 248: 31 249: 83 250: 5

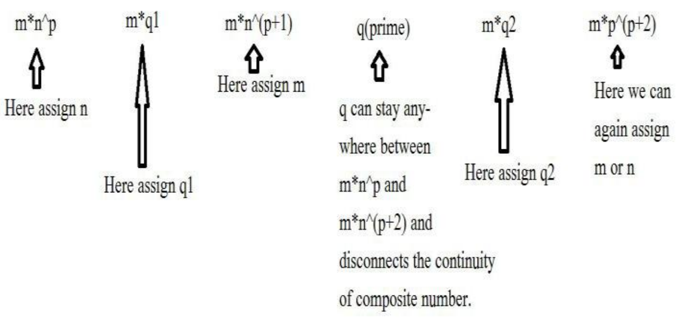
The problem is to prove the conjecture, or find a counter-example.

Solution

Case 1 :
If we can prove there is at least one prime between n^p and n^(p+1) then we can assign n, to
both n^p and n^(p+1) as the image below says :



Case 2 :
If we can prove there is at least one prime between m*n^P and m*n^(p+2) then we can assign
n to m*n^p , m to m*n(p+1) and again n or m to n^(p+2) as the below picture says :



Now,

1) Between 2 and 2^2 there is a prime 3
2) Between 2^2 and 2^3 there is prime 5, 7.
3) Between 3 and 3^2 there is prime 5, 7.
4) Onwards i.e. between 2^3 and 2^4 or 3^2 and 3^3 .... there are consecutive square numbers. According to Legendre's conjecture (which have been proved above) there is at least one prime.
5) Now Case 2 is trivial from Case 1.

So, Grimm's conjecture is true.

IV. Balanced Prime problem

A **balanced prime** is a prime number that is equal to the arithmetic mean of the nearest primes above and below. Or to put it algebraically, given a prime number p_n, where *n* is its index in the ordered set of prime numbers,

$$p_n = \frac{p_{n-1} + p_{n+1}}{2}.$$

The first few balanced primes are

5, 53, 157, 173, 211, 257, 263, 373, 563, 593, 607, 653, 733, 947, 977, 1103 (sequence A006562 in OEIS).

For example, 53 is the sixteenth prime. The fifteenth and seventeenth primes, 47 and 59, add up to 106, half of which is 53, thus 53 is a balanced prime.

When 1 was considered a prime number, 2 would have correspondingly been considered the first balanced prime since

$$2 = \frac{1+3}{2}.$$

It is conjectured that there are infinitely many balanced primes.

Problem source :
http://en.wikipedia.org/wiki/Balanced_prime

Solution

Let there are finite number of balanced primes.
We are considering primes of the form p-12, p-6, p.
Now let's $\mathrm{p_n}$ -6 be the last balanced prime.
Now we form odd number table as below :
Any one column is divisible by 3. Let the left most column is divisible by 3. *Italic* numbers are divisible by 5 and Underlined numbers are divisible by 7.

$\mathrm{p_n}$ -10	p_n -8	$\mathrm{p_n}$ -6
$\mathrm{p_n}$ -4	$\underline{\mathrm{p_n}\text{ -2}}$	$\mathrm{p_n}$
$\mathrm{p_n}$ +2	$\mathrm{p_n}$ +4	$\mathrm{p_n}$ +6
$\mathrm{p_n}$ +8	$\mathrm{p_n}$ +10	p_n +12
$\mathrm{p_n}$ +14	$\mathrm{p_n}$ +16	$\mathrm{p_n}$ +18
$\mathrm{p_n}$ +20	p_n +22	$\mathrm{p_n}$ +24
$\mathrm{p_n}$ +26	$\mathrm{p_n}$ +28	$\mathrm{p_n}$ +30
$\mathrm{p_n}$ +32	$\mathrm{p_n}$ +34	$\mathrm{p_n}$ +36
$\mathrm{p_n}$ +38	$\underline{\mathrm{p_n}\text{ +40}}$	p_n +42
$\mathrm{p_n}$ +44	$\mathrm{p_n}$ +46	$\mathrm{p_n}$ +48
$\mathrm{p_n}$ +50	p_n +52	$\underline{\mathrm{p_n}\text{ +54}}$
$\mathrm{p_n}$ +56	$\mathrm{p_n}$ +58	$\mathrm{p_n}$ +60
$\mathrm{p_n}$ +62	$\mathrm{p_n}$ +64	$\mathrm{p_n}$ +66
$\mathrm{p_n}$ +68	$\mathrm{p_n}$ +70	p_n +72
$\mathrm{p_n}$ +74	$\mathrm{p_n}$ +76	$\mathrm{p_n}$ +78
$\mathrm{p_n}$ +80	$\underline{p_n\text{ +82}}$	$\mathrm{p_n}$ +84
$\mathrm{p_n}$ +86	$\mathrm{p_n}$ +88	$\mathrm{p_n}$ +90
$\mathrm{p_n}$ +92	$\mathrm{p_n}$ +94	$\underline{\mathrm{p_n}\text{ +96}}$
$\mathrm{p_n}$ +98	$\mathrm{p_n}$ +100	p_n +102
$\mathrm{p_n}$ +104	$\mathrm{p_n}$ +106	$\mathrm{p_n}$ +108

We see that $\underline{\mathrm{p_n}\text{ +96}}$ and p_n +102 are composite and forming a balanced prime sets viz. $\mathrm{p_n}$ +94, $\mathrm{p_n}$ +100 and $\mathrm{p_n}$ +106.
Now according to our assumption one of this must be divisible by any prime before it i.e. one of them have to be composite.
Now, we will find infinite number of balanced prime sets only by 5 and 7 and it will occur in a regular frequency i.e. after every 70 numbers and in a particular column it will occur after every 210 numbers.
There are only finite number of primes before $\mathrm{p_n}$.

- ⇨ They cannot make every balanced prime set unbalanced by dividing any of three primes forming the balanced prime set.

So, there needs to be born primes after $\mathrm{p_n}$ so that every regular occurrence of balanced prime set by 5,7 can be made unbalanced by dividing with those primes at least one of the prime forming balanced set.
But there is no regular occurrence of prime whereas 5,7 will be forming balanced set of primes in a regular manner.

- ⇨ There will be a shortcoming somewhere of primes to make each and every balanced pair formed by 5,7 in a regular manner.
- ⇨ There will be balanced prime after $\mathrm{p_n}$.

Here is the contradiction.

- ⇨ Balanced primes are infinite.

Proved.

V. Prime Triplet problem

In mathematics, a **prime triplet** is a set of three prime numbers of the form (p, p + 2, p + 6) or (p, p + 4, p + 6). With the exceptions of (2, 3, 5) and (3, 5, 7), this is the closest possible grouping of three prime numbers, since every third odd number greater than 3 is divisible by 3, and hence not prime.

The first prime triplets (sequence A098420 in OEIS) are

(5, 7, 11), (7, 11, 13), (11, 13, 17), (13, 17, 19), (17, 19, 23), (37, 41, 43), (41, 43, 47), (67, 71, 73), (97, 101, 103), (101, 103, 107), (103, 107, 109), (107, 109, 113), (191, 193, 197), (193, 197, 199), (223, 227, 229), (227, 229, 233), (277, 281, 283), (307, 311, 313), (311, 313, 317), (347, 349, 353), (457, 461, 463), (461, 463, 467), (613, 617, 619), (641, 643, 647), (821, 823, 827), (823, 827, 829), (853, 857, 859), (857, 859, 863), (877, 881, 883), (881, 883, 887)

A prime triplet contains a pair of twin primes (p and p + 2, or p + 4 and p + 6), a pair of cousin primes (p and p + 4, or p + 2 and p + 6), and a pair of sexy primes (p and p + 6).

A prime can be a member of up to three prime triplets - for example, 103 is a member of (97, 101, 103), (101, 103, 107) and (103, 107, 109). When this happens, the five involved primes form a prime quintuplet.

A prime quadruplet (p, p + 2, p + 6, p + 8) contains two overlapping prime triplets, (p, p + 2, p + 6) and (p + 2, p + 6, p + 8).

Similarly to the twin prime conjecture, it is conjectured that there are infinitely many prime triplets. The first known gigantic prime triplet was found in 2008 by Norman Luhn and François Morain. The primes are (p, p + 2, p + 6) with p = 2072644824759 × 2^{33333} − 1. As of May 2013 the largest known prime triplet contains primes with 16737 digits and was found by Peter Kaiser. The primes are (p, p + 4, p + 6) with p = 6521953289619 × 2^{55555} − 5.

Problem source : http://en.wikipedia.org/wiki/Prime_triplet

Solution

One of the column is divisible by 3. Let the left most column is divisible by 3. We will mark other composite numbers by underline.
Let's say p_n-6, p_n-2 and p_n form the last triplet prime set. We assume there is finite number of prime triplet set.
After p_n one of every triplet set should get divided by prime before it.

p_n-10 p_n-8 p_n-6
p_n-4 p_n-2 p_n
p_n+2 $\underline{p_n+4}$ $\underline{p_n+6}$
p_n+8 p_n+10 p_n+12
p_n+14 $\underline{p_n+16}$ $\underline{p_n+18}$
p_n+20 p_n+22 p_n+24
p_n+26 $\underline{p_n+28}$ $\underline{p_n+30}$
p_n+32 p_n+34 p_n+36
p_n+38 $\underline{p_n+40}$ $\underline{p_n+42}$
p_n+44 p_n+46 p_n+48
p_n+50 $\underline{p_n+52}$ $\underline{p_n+54}$
p_n+56 p_n+58 p_n+60
p_n+62 $\underline{p_n+64}$ $\underline{p_n+66}$
p_n+68 p_n+70 p_n+72
p_n+74 $\underline{p_n+76}$ $\underline{p_n+78}$
p_n+80 p_n+82 p_n+84
p_n+86 $\underline{p_n+88}$ $\underline{p_n+90}$
p_n+92 p_n+94 p_n+96
p_n+98 $\underline{p_n+100}$ $\underline{p_n+102}$
p_n+104 p_n+106 p_n+108

As we can see the primes occur in a regular manner if we mark the composite number as above. But primes don't have any regular pattern to generate.
Here is the contradiction. So this case cannot happen.
Again we form the table as below :
p_n-10 p_n-8 p_n-6
p_n-4 p_n-2 p_n
p_n+2 $\underline{p_n+4}$ p_n+6
p_n+8 p_n+10 $\underline{p_n+12}$
p_n+14 $\underline{p_n+16}$ p_n+18
p_n+20 p_n+22 $\underline{p_n+24}$
p_n+26 $\underline{p_n+28}$ p_n+30
p_n+32 p_n+34 $\underline{p_n+36}$
p_n+38 $\underline{p_n+40}$ p_n+42
p_n+44 p_n+46 $\underline{p_n+48}$
p_n+50 $\underline{p_n+52}$ p_n+54
p_n+56 p_n+58 $\underline{p_n+60}$
p_n+62 $\underline{p_n+64}$ p_n+66
p_n+68 p_n+70 $\underline{p_n+72}$
p_n+74 $\underline{p_n+76}$ p_n+78
p_n+80 p_n+82 $\underline{p_n+84}$
p_n+86 $\underline{p_n+88}$ p_n+90
p_n+92 p_n+94 $\underline{p_n+96}$
p_n+98 $\underline{p_n+100}$ p_n+102
p_n+104 p_n+106 $\underline{p_n+108}$

As we can see the primes occur in a regular manner if we mark the composite number as above. But primes don't have any regular pattern to generate.
Here is the contradiction. So this case cannot happen.

- ⇨ The composite number should occur in an awkward pattern
- ⇨ More primes are necessary to make them composite as for each prime occurrence of the multiples of the primes are regular but primes don't occur in regular pattern.

Again we form the table as below :
p_n-10 p_n-8 p_n-6
p_n-4 p_n-2 p_n
p_n+2 $\underline{p_n+4}$ p_n+6
p_n+8 $\underline{p_n+10}$ p_n+12
p_n+14 p_n+16 $\underline{p_n+18}$
p_n+20 $\underline{p_n+22}$ p_n+24
p_n+26 p_n+28 $\underline{p_n+30}$
p_n+32 p_n+34 $\underline{p_n+36}$
p_n+38 $\underline{p_n+40}$ p_n+42
p_n+44 p_n+46 $\underline{p_n+48}$
p_n+50 $\underline{p_n+52}$ p_n+54
p_n+56 p_n+58 $\underline{p_n+60}$
p_n+62 $\underline{p_n+64}$ p_n+66
p_n+68 $\underline{p_n+70}$ p_n+72
p_n+74 $\underline{p_n+76}$ p_n+78
p_n+80 p_n+82 $\underline{p_n+84}$
p_n+86 $\underline{p_n+88}$ p_n+90
p_n+92 p_n+94 $\underline{p_n+96}$
p_n+98 p_n+100 $\underline{p_n+102}$
p_n+104 p_n+106 $\underline{p_n+108}$

Not to form any prime triplet after p_n => there must be a composite number in each row.
Now, before p_n the number of primes is less than p_n.
Now, after p_n next multiple of p_n will occur in the same column is $6p_n$.
Between p_n and $6p_n$ there are p_n number of rows.
But before p_n there are less than p_n number of primes which can make composite one number of each row.

- ⇨ There will be primes left in a row.
- ⇨ Triplet prime set is there after p_n.
- ⇨ Triplet prime set is infinite.

Proved.

VI. COROLLARY

Twin primes are infinite as in the case of twin prime also each row should have at least one composite number.
Polignac's conjecture :

In number theory, **Polignac's conjecture** was made by Alphonse de Polignac in 1849 and states:

> For any positive even number *n*, there are infinitely many prime gaps of size *n*. In other words: There are infinitely many cases of two consecutive prime numbers with difference *n*.

The conjecture has not yet been proven or disproven for a given value of n. In 2013 an important breakthrough was made by Zhang Yitang who proved that there are infinitely many prime gaps of size n for some value of n < 70,000,000.[2]

For n = 2, it is the twin prime conjecture. For n = 4, it says there are infinitely many cousin primes (p, p + 4). For n = 6, it says there are infinitely many sexy primes (p, p + 6) with no prime between p and p + 6.

Problem source : http://en.wikipedia.org/wiki/Polignac%27s_conjecture

Solution

For twin prime see the corollary of **Triplet Prime** problem. (End of Solution of **Triplet Prime**)
Now, we will prove there are infinitely many cousin primes.
Let, the series of cousin primes is finite.
Let, p_n – 6 and p_n -2 form the last cousin prime pair.
Any one column is divisible by 3. Let the left most column is divisible by 3. We will mark other composite numbers by underline.
Now we form the table as below :

p_n-4	p_n-2	$\underline{p_n}$
p_n+2	p_n+4	$\underline{p_n+6}$
p_n+8	p_n+10	$\underline{p_n+12}$
p_n+14	p_n+16	$\underline{p_n+18}$
p_n+20	p_n+22	$\underline{p_n+24}$
p_n+26	p_n+28	$\underline{p_n+30}$
p_n+32	p_n+34	$\underline{p_n+36}$
p_n+38	p_n+40	$\underline{p_n+42}$
p_n+44	p_n+46	$\underline{p_n+48}$
p_n+50	p_n+52	$\underline{p_n+54}$
p_n+56	p_n+58	$\underline{p_n+60}$
p_n+62	p_n+64	$\underline{p_n+66}$
p_n+68	p_n+70	$\underline{p_n+72}$
p_n+74	p_n+76	$\underline{p_n+78}$
p_n+80	p_n+82	$\underline{p_n+84}$
p_n+86	p_n+88	$\underline{p_n+90}$
p_n+92	p_n+94	$\underline{p_n+96}$
p_n+98	p_n+100	$\underline{p_n+102}$
p_n+104	p_n+106	$\underline{p_n+108}$

The condition to satisfy that there is no cousin prime pair after p_n -2 is one of the two columns should be composite (except the column divisible by 3).

- ⇨ The primes occur in a regular fashion.
- ⇨ Here is the contradiction.

Now, to deny the case there needs to be composite number in the middle column in an irregular fashion.
Now, next p_n -2 occurs in the middle column after p_n -2 rows.

- ⇨ There are more than p_n -2 composite numbers before next p_n – 2 occurs in the table.
- ⇨ But before p_n -2 there are less than p_n -2 number of primes.
- ⇨ This case cannot happen.
- ⇨ There are infinite number of Cousin primes.

Proved.
The conclusion can be done in other way also :
All the number of third column is composite. But there is at least difference of 2 between any consecutive prime.

- ⇨ *There will be numbers left between the series of multiples of prime before p_n -2.*
- ⇨ *There will be Cousin prime generated after p_n -2.*

Here is the contradiction.

- ⇨ *There are infinite number of Cousin primes.*

Proved.
Now we will prove there are infinite number of sexy primes.
Let, the series of sexy primes is finite.
Let, p_n – 8 and p_n -2 form the last sexy prime pair.
One of the column is divisible by 3. Let the left most column is divisible by 3. We will mark other composite numbers by underline.
Now we form the table as below :

p_n-4	p_n-2	$\underline{p_n}$
p_n+2	$\underline{p_n+4}$	p_n+6
p_n+8	p_n+10	$\underline{p_n+12}$
p_n+14	$\underline{p_n+16}$	p_n+18
p_n+20	p_n+22	$\underline{p_n+24}$
p_n+26	$\underline{p_n+28}$	p_n+30
p_n+32	p_n+34	$\underline{p_n+36}$
p_n+38	$\underline{p_n+40}$	p_n+42
p_n+44	p_n+46	$\underline{p_n+48}$
p_n+50	$\underline{p_n+52}$	p_n+54
p_n+56	p_n+58	$\underline{p_n+60}$
p_n+62	$\underline{p_n+64}$	p_n+66
p_n+68	p_n+70	$\underline{p_n+72}$
p_n+74	$\underline{p_n+76}$	p_n+78
p_n+80	p_n+82	$\underline{p_n+84}$
p_n+86	$\underline{p_n+88}$	p_n+90
p_n+92	p_n+94	$\underline{p_n+96}$
p_n+98	$\underline{p_n+100}$	p_n+102
p_n+104	p_n+106	$\underline{p_n+108}$

The condition to satisfy that there is no sexy prime pair after p_n -2 is alternate number in each column should be composite (except the column divisible by 3).

- ⇨ The primes occur in a regular fashion.
- ⇨ Here is the contradiction.

Now, to deny the case there needs to be more composite numbers.
Now, next p_n -2 occurs in the middle column after p_n -2 rows.

- ⇨ There are more than p_n -2 composite numbers before next p_n – 2 occurs in the table.
- ⇨ But before p_n -2 there are less than p_n -2 number of primes.
- ⇨ This case cannot happen.
- ⇨ There are infinite number of Sexy primes.

Proved.

Problem

Can a prime p satisfy $2^{p-1} \equiv 1 \pmod{p^2}$ and $3^{p-1} \equiv 1 \pmod{p^2}$ simultaneously?

Solution

Let p be the prime which simultaneously satisfy both the equation.

- ⇨ 2^(p-1) – 1 = mp² ....(1)
- ⇨ 3^(p-1) – 1 = np² ....(2)

Now dividing both sides of equation (1) by 8 we get,
2^(p-1) – 1 = (8m-1)p² (as LHS ≡ -1 and p² ≡ 1 (mod 8) and putting m = 8m-1) .... (3)

Now dividing both sides of equation (2) by 8 we get,
$3^{(p-1)} - 1 = 8np^2$ (as LHS ≡ 0 and $p^2 \equiv 1 \pmod 8$) and putting n = 8n) ....(4)
Now, dividing both sides of equation (3) by 3 we get,
$2^{(p-1)} - 1 = (24m+15)p^2$ (as LHS ≡ 0 and $p^2 \equiv 1 \pmod 3$) so 8m-1 must be ≡ 0 (mod 3), so putting m = 3m+2) ....(5)
Now dividing both sides of equation (4) by 3 we get,
$3^{(p-1)} - 1 = 8(3n+1)p^2$ (as LHS ≡ -1 and $p^2 \equiv 1 \pmod 3$), 8n must be ≡ -1 (mod 3), so putting 8n = 8(3n+1)) ....(6)
Now, dividing both sides of equation (6) by 16 we get,
LHS ≡ 0 or 8 (mod 16)
Case 1 : LHS ≡ 8 (mod 16)

- ⇨ p – 1 = 2k where k is odd.
- ⇨ p – 1 = 2(2k+1) (putting 2k+1 in place of k as k is odd)
- ⇨ p = 4k+3
- ⇨ $p^2 = 16k^2 + 24k + 9$
- ⇨ $8p^2 = 128k^2 + 192k + 72$
- ⇨ $8p^2 \equiv 8 \pmod{16}$
- ⇨ 3n+1 must be ≡ 1 (mod 16)
- ⇨ n must be of the form 16n

Equation (6) becomes, $3^{(p-1)} - 1 = 8(48n+1)p^2$ ....(7)
Now, dividing both sides of equation (5) by 16 we get,
LHS ≡ -1 (mod 16)
$p^2 \equiv 1$ or 9 (mod 16)
Case 1a : $p^2 \equiv 1 \pmod{16}$
Now, $p^2 = 16k^2 + 24k + 9$

- ⇨ k is odd as $p^2 \equiv 1 \pmod{16}$
- ⇨ $p^2 = 16(2k+1)^2 + 24(2k+1) + 9$ (putting k = 2k+1 as k is odd)
- ⇨ $p^2 = 64k^2 + 112k + 49$
- ⇨ p = 8k + 7

Now, 24m+15 must be ≡ -1 (mod 16)

- ⇨ m is even.

Equation (5) becomes, $2^{(p-1)} - 1 = (48m+15)p^2$ (putting 2m in place of m as m is even) ....(8)
Now, dividing both sides of equation (7) by 32,
$3^{(p-1)} - 1 = 3^{(8k+7-1)} - 1 = 3^{(8k+6)} - 1 = (3^{8k})(3^6) - 1$
Now, $3^{8k} \equiv 1 \pmod{32}$ (as (any odd number)^8 ≡ 1 (mod 32))
$3^6 \equiv 25 \pmod{32}$

- ⇨ LHS = $(3^{8k})(3^6) - 1 \equiv 1*25 - 1 \equiv 24 \pmod{32}$

Now, we have, $p^2 = 64k^2 + 112k + 49$

- ⇨ $8p^2 = 512k^2 + 896k + 392$
- ⇨ $8p^2 \equiv 8 \pmod{32}$

Now, 48n+1 ≡ 1 (mod 32) if n is even & 48n+1 ≡ 17 (mod 32) if n is odd.
When n is even, RHS ≡ 8*1 ≡ 8 (mod 32)
When n is odd, RHS ≡ 17*8 ≡ 8 (mod 32)
But LHS ≡ 24 (mod 32)
Here is the contradiction.
Case 1b : $p^2 \equiv 9 \pmod{16}$
Now, $p^2 = 16k^2 + 24k + 9$

- ⇨ k is even.
- ⇨ $p^2 = 16(2k)^2 + 24(2k) + 9$ (putting 2k in place of k as k is even)
- ⇨ $p^2 = 64k^2 + 48k + 9$
- ⇨ p = 8k + 3

Now, dividing both sides of equation (5) by 16 we get,
LHS ≡ -1 (mod 16)
Now, 24m+15 ≡ 15 (mod 16) if m is even & 24m+15 ≡ 7 (mod 16) if m is odd
When m is even, RHS = $(24m+15)p^2 \equiv 15*9 \equiv 7 \pmod{16}$
When m is odd, RHS = $(24m+15)p^2 \equiv 7*9 \equiv -1 \pmod{16}$

- ⇨ m is odd.

Equation (5) becomes, $2^{(p-1)} - 1 = (48m+39)p^2$ (putting 2m+1 in place of m as m is odd)...........(9)
Now, dividing both sides of equation (7) by 32 we get,
LHS = $3^{(p-1)} - 1 = 3^{(8k+3-1)} - 1 = (3^{8k})(3^2) - 1 \equiv 1*9 - 1 \equiv 8 \pmod{32}$
Now, we have, $p^2 = 64k^2 + 48k + 9$

- ⇨ $8p^2 = 512k^2 + 384k + 72$
- ⇨ $8p^2 \equiv 8 \pmod{32}$
- ⇨ 48n+1 must be ≡ 1 (mod 32)
- ⇨ n is even.

Equation (7) becomes, $3^{(p-1)} - 1 = 8(96n+1)p^2$ (putting 2n in place of n as n is even) ....(10)
Now, dividing both sides of equation (9) by 32 we get,
LHS ≡ -1 (mod 32)
$p^2 \equiv 9$ or 25 (mod 32)
Case 1b(i) : $p^2 \equiv 9 \pmod{32}$

- ⇨ k is even.
- ⇨ $p^2 = 64(2k)^2 + 48(2k) + 9$ (putting 2k in place of k as k is even)
- ⇨ $p^2 = 256k^2 + 96k + 9$
- ⇨ p = 16k+3

Now, 48m + 39 ≡ 7 (mod 32) if m is even and ≡ 23 (mod 32) if m is odd.

- ⇨ m must be even.

Equation (9) becomes, $2^{(p-1)} - 1 = (96m+39)p^2$ (putting 2m in place of m as m is even).........(11)
Now, dividing both sides of equation (10) by 64 we get,
LHS = $3^{(p-1)} - 1 = 3^{(16k+3-1)} - 1 = (3^{16k})(3^2) - 1 \equiv 1*9 - 1 \equiv 8 \pmod{32}$ (as (any odd power)^{16} ≡ 1 (mod 64))
We have, $p^2 = 256k^2 + 96k + 9$

- ⇨ $8p^2 = 2048p^2 + 768p + 72$
- ⇨ $8p^2 \equiv 8 \pmod{64}$

Now, 96n+1 must be ≡ 1 (mod 64) => n is even.
Equation (10) becomes, $3^{(p-1)} - 1 = 8(192n+1)p^2$ (putting 2n in place of n as n is even).......(12)
Now, dividing both sides of equation (11) by 64 we get,
LHS ≡ -1 (mod 64)
$p^2 \equiv 9$ or 41 (mod 64)
Case 1b(i)/(a) : $p^2 \equiv 9 \pmod{64}$

- ⇨ k is even.
- ⇨ $p^2 = 256(2k)^2 + 96(2k) + 9$ (putting 2k in place of k as k is even)
- ⇨ $p^2 = 1024k^2 + 192k + 9$
- ⇨ p = 32k + 3

96m+39 ≡ 7 if m is odd and ≡ 39 if m is even (mod 64)
According to our case, 96m+39 must be ≡ 7 (mod 64)
Because then, RHS ≡ 9*7 ≡-1 (mod 64)

Equation (11) becomes, 2^(p-1) – 1 = (192m+135)p² (putting 2m+1 in place of m as m is odd)........(13)
Now, dividing both sides of equation (12) by 128 we get,
LHS = 3^(p-1) – 1 = 3^(32k+3-1) – 1 = (3^32k)(3²) – 1 ≡ 1*9 – 1 ≡ 8 (mod 128) (as (any odd number)^32 ≡ 1 (mod 128))
We have, p² = 1024k² + 192k + 9

- ⇨ 8p² = 8192p² + 1536k + 72
- ⇨ 8p² ≡ 72 (mod 128)

Now, 192n+1 ≡ 1 if n is even and ≡ 65 if n is odd (mod 32)
We see that in both the cases, RHS ≡ 72 (mod 128)
Here is the contradiction.
Case 1b(i)/(b) : p² ≡ 41 (mod 64)

- ⇨ p² = 256(2k+1)² + 96(2k+1) + 9
- ⇨ p² = 1024k²+ 1216k + 361
- ⇨ p = 32k+19

Now, dividing both sides of equation (11) by 64 we get,
LHS ≡ -1 (mod 64)
p² ≡ 41 (mod 64)

- ⇨ 96m+39 must be ≡ 39 (mod 64) (as 41*39 ≡ -1 (mod 32)
- ⇨ m is even.

Equation (11) becomes, 2^(p-1) – 1 = (192m+39)p² (putting 2m in place of m as m is even) ..... (14)
Now, dividing both sides of equation (12) by 128 we get,
LHS = 3^(p-1) – 1 = 3^(32k+19-1) – 1 = (3^32k)(3^18) – 1 ≡ 1*73 – 1 ≡ 72 (mod 128)
We have, p² = 1024k²+ 1216k + 361

- ⇨ 8p² = 8192k² + 9728k + 2888
- ⇨ 8p² ≡ 72 (mod 128)
- ⇨ 192n+1 must be ≡ 1 (mod 128)
- ⇨ n is even.

Equation (12) becomes, 3^(p-1) – 1 = 8(384n+1)p² (putting 2n in place of n as n is even) ......(15)
Now, dividing both sides of equation (14) by 256 we get,
LHS ≡ -1 (mod 256)
We have p² = 1024k² + 1216k + 361

- ⇨ p² ≡ 169 or 233 or 297 (mod 256)

Now, 192m+39 ≡ 231 or 167 or 103 (mod 256)
We see that in this way the cases will go on increasing the problem giving ultimately no solution. So we can conclude that there is no prime p which satisfy $2^{p-1} \equiv 1 \pmod{p^2}$ and $3^{p-1} \equiv 1 \pmod{p^2}$ simultaneously.

17 tips with proof to solve problems on number theory :

1. *(Any odd number)² ≡ 1 (mod 4)*

Proof : Let a be any odd number.

- ⇨ a = 2n±1
- ⇨ a² = (2n±1)²
- ⇨ a² = 4n² ± 4n + 1
- ⇨ a² = 4n(n±1) + 1
- ⇨ a² ≡ 1 (mod 4)

Proved.

2. *(Any odd number)² ≡ 1 (mod 8)*

Proof : Let a be any odd number.

- ⇨ a = 2n±1
- ⇨ a² = (2n±1)²
- ⇨ a² = 4n² ± 4n + 1
- ⇨ a² = 4n(n±1) + 1

Now if n is odd then (n±1) is even; if n is even then (n±1) is odd.

- ⇨ n(n±1) is divisible by 2.
- ⇨ 4n(n±1) is divisible by 8
- ⇨ a² = 4n(n±1) + 1 ≡ 1 (mod 8)

Proved.

3. *(Any odd number)² ≡ 1 or 9 (mod 16)*

We have already proved (any odd number)² ≡ 1 (mod 8)
Let a be any odd number.
Then a² = 8n+1
If n is even then 8n is divisible by 16.

- ⇨ a² = 8n + 1 ≡ 1 (mod 16)

If n is odd then 8n = 8(2n+1) = 16n + 8 (putting 2n +1 in place of n as n is odd)

- ⇨ 8(2n+1) = 16n+8 ≡ 8 (mod 16)
- ⇨ a² ≡ 8(2n+1) + 1 ≡ 8 + 1 = 9 (mod 16)
- ⇨ a² ≡ 1 or 9 (mod 16)

Proved.

4. *(Any odd number)⁴ ≡ 1 (mod 16)*

We have already proved, (any odd number)² ≡ 1 (mod 8)
Let a be any odd number.

- ⇨ a² = 8n+1
- ⇨ a⁴ = (8n+1)²
- ⇨ a⁴ = 64n² + 16n + 1
- ⇨ a⁴ = 16(4n²+1) + 1
- ⇨ a⁴ ≡ 1 (mod 16)

Proved.

5. *(Any odd number)² ≡ 1 or 9 or -7 or -15 (mod 32)*

We have already proved, (any odd number)² ≡ 1 or 9 (mod 16)
Let a be any odd number.

- ⇨ a² = 16n+1 or 16m+9

Taking a² = 16n+1.
If n is even then 16n is divisible by 32.

- ⇨ a² = 16n+1 ≡ 1 (mod 32)

If n is odd then a² = 16(2n+1) +1 (putting 2n+1 in place of n as n is odd)

- ⇨ a² = 32n + 17
- ⇨ a² ≡ 17 (mod 32)
- ⇨ a² ≡ 32 -15 (mod 32)
- ⇨ a² ≡ -15 (mod 32)

Taking a² = 16m+9
If m is even, then 16m is divisible by 32.

- ⇨ a² = 16m+9 ≡ 9 (mod 32)

If m is odd then a² = 16(2m+1) +9 (putting 2m+1 in place of m as m is odd)

- ⇨ a² = 32m + 25
- ⇨ a² ≡ 25 (mod 32)
- ⇨ a² ≡ 32 – 7 (mod 32)
- ⇨ a² ≡ -7 (mod 32)

So, a² ≡ 1 or 9 or -7 or -15

Proved.

6. *(Any odd number)^4 ≡ 1 or -15 (mod 32*

We have already proved, (any odd number)^4 ≡ 1 (mod 16)
Let a be any odd number.
Then $a^4 = 16n+1$
If n is even then 16n+1 is divisible by 32.
⇨ $a^4 = 16n+1 \equiv 1 \pmod{32}$

If n is odd then $a^4 = 16(2n+1)+1$ (putting 2n+1 in place of n as n is odd)
⇨ $a^4 = 32n+17$
⇨ $a^4 \equiv 17 \pmod{32}$
⇨ $a^4 \equiv 32 -15 \pmod{32}$
⇨ $a^4 \equiv -15 \pmod{32}$

So, $a^4 \equiv 1$ or $-15 \pmod{32}$
Proved.

7. *(Any odd number)^8 ≡ 1 (mod 32)*

We have already proved (any odd number)^4 ≡ 1 (mod 16)
Let a be any odd number.
⇨ $a^4 = 16n+1$
⇨ $a^8 = (16n+1)^2$
⇨ $a^8 = 256n^2 + 32n + 1$
⇨ $a^8 = 32n(8n+1) + 1$
⇨ $a^8 \equiv 1 \pmod{32}$

Proved.

8. *(Any odd number)^2 ≡ 1 or 9 or 25 or -15 or 17 or -7 or -23 or -31 (mod 64)*

We have already proved, (any odd number)^2 ≡ 1 or 9 or -7 or -15 (mod 32)
⇨ $a^2 = 32n+1$ or $32m+9$ or $32p-7$ or $32q -15$.

Taking $a^2 = 32n+1$
If n is even then $a^2 = 32(2n) + 1$ (putting 2n in place of n as n is even)
⇨ $a^2 = 64n + 1$
⇨ $a^2 \equiv 1 \pmod{32}$

If n is odd then $a^2 = 32(2n+1) +1$ (putting 2n+1 in place of n as n is odd)
⇨ $a^2 = 64n + 33 \pmod{64}$
⇨ $a^2 \equiv 33 \pmod{64}$
⇨ $a^2 \equiv 64 - 31 \pmod{64}$
⇨ $a^2 \equiv -31 \pmod{64}$

Taking $a^2 = 32m+9$
If m is even then $a^2 = 32(2m)+9$ (putting 2m in place of m as m is even)
⇨ $a^2 = 64m + 9$
⇨ $a^2 \equiv 9 \pmod{64}$

If m is odd then $a^2 = 32(2m+1) + 9$ (putting 2m+1 in place of m as m is odd)
⇨ $a^2 = 64m + 41$
⇨ $a^2 \equiv 41 \pmod{64}$
⇨ $a^2 \equiv 64 - 23 \pmod{64}$
⇨ $a^2 \equiv -23 \pmod{64}$

Taking $a^2 = 32p -7$
If p is even then $a^2 = 32(2p) - 7$ (putting 2p in place of p as p is even)
⇨ $a^2 = 64p - 7$
⇨ $a^2 \equiv - 7 \pmod{64}$

If p is odd then $a^2 = 32(2p+1) - 7$ (putting 2p+1 in place of p as p is odd)
⇨ $a^2 = 64p + 25$
⇨ $a^2 \equiv 25 \pmod{64}$

Taking $a^2 = 32q - 15$
If q is even then $a^2 = 32(2q) - 15$
⇨ $a^2 = 64q - 15$
⇨ $a^2 \equiv -15 \pmod{64}$

If q is odd then $a^2 = 32(2q+1) - 15$ (putting 2q+1 in place of q as q is odd)
⇨ $a^2 = 64q + 17$
⇨ $a^2 \equiv 17 \pmod{64}$

So, $a^2 \equiv 1$ or 9 or 25 or -15 or 17 or -7 or -23 or -31 (mod 64)
Proved.

9. *(Any odd number)^4 ≡ 1 or 17 or -31 or -15 (mod 64)*

We have already proved (any odd number)^4 ≡ 1 or -15 (mod 32)
Let a be any odd number.
⇨ $a^4 = 32n+1$ or $32m-15$.

Taking, $a^4 = 32n+1$
If n is even then $a^4 = 32(2n)+1$ (putting 2n in place of n as n is even)
⇨ $a^4 = 64n+1$
⇨ $a^4 \equiv 1 \pmod{64}$

If n is odd then $a^4 = 32(2n+1)+1$ (putting 2n+1 in place of n as n is odd)
⇨ $a^4 = 64n + 33 \pmod{64}$
⇨ $a^4 \equiv 33 \pmod{64}$
⇨ $a^4 \equiv 64 - 31 \pmod{64}$
⇨ $a^4 \equiv -31 \pmod{64}$

Taking, $a^4 = 32m - 15$
If m is even then $a^4 = 32(2m) - 15$ (putting 2m in place of m as m is even)
⇨ $a^4 = 64m - 15$ 33
⇨ $a^4 \equiv - 15 \pmod{64}$

If m is odd then $a^4 \equiv 32(2m+1) - 15$ (putting 2m+1 in place of m as m is odd)
⇨ $a^4 \equiv 64m + 17$
⇨ $a^4 \equiv 17 \pmod{64}$

So, $a^4 \equiv 1$ or 17 or -31 or -15 (mod 64)
Proved.

10. *(Any odd number)^8 ≡ 1 or -31 (mod 64)*

We have already proved (any odd number)^8 ≡ 1 (mod 32)
Let a be any odd number.
⇨ $a^8 = 32n+1$

If n is even then $a^8 = 32(2n)+1$ (putting 2n in place of n as n is even)

- ⇨ $a^8 = 64n+1$
- ⇨ $a^8 \equiv 1 \pmod{64}$

If n is odd then $a^8 = 32(2n+1)+1$ (putting 2n+1 in place of n as n is odd)

- ⇨ $a^8 = 64n+33$
- ⇨ $a^8 \equiv 33 \pmod{64}$
- ⇨ $a^8 \equiv 64 - 31 \pmod{64}$
- ⇨ $a^8 \equiv -31 \pmod{64}$

So, $a^8 \equiv 1$ or $-31 \pmod{64}$
Proved.

11. (Any odd number)^16 ≡ 1 (mod 64)

We have already proved, (any odd number)$^8 \equiv 1 \pmod{32}$
Let a be any odd number.
Then $a^8 = 32n+1$

- ⇨ $a^{16} = (32n+1)^2$
- ⇨ $a^{16} = 1024n^2+64n+1$
- ⇨ $a^{16} = 64n(16n+1)+1$
- ⇨ $a^{16} \equiv 1 \pmod{16}$

Proved.

12. The square of any odd integer can be written as difference of square of two consecutive integers. The consecutive integers are of the form 2n(n-1) and 2n(n-1)+1 where n is nth odd integer.

For example, 11 is 6th odd integer.
Now, 2n(n-1) = 2*6*(6-1) = 60
Therefore, 11^2 = 61^2 - 60^2
or, 11^2 + 60^2 = 61^2.

13. The square of any even integer can be written as either difference of two consecutive odd integers or two consecutive even integers. The consecutive odd/even integers are of the form n^2-1 and n^2+1 where n is n-th even integer considering 2 as first even integer.

For example, 8 is 4th even integer.
Now n^2-1 = 4^2-1 = 15 and n^2+1 = 17
Therefore, 8^2 = 17^2 - 15^2
or, 8^2 + 15^2 = 17^2.

14. How to find Pythagorian triplet ?

1. Choose a composite number.
2. Now square it.
3. Now factorize it.
4. Now divide it in two factors such that both factors are either even or both are odd.
5. Now, find the middle number between the two.
6. Now find the difference betweem the two factors and divide it by 2.
7. Now, (original number)^2 + (number found in step 6)^2 = (middle number found in step 5)^2

For example :
1. Lets take 14.
2. 196
3. 2*2*7*7
4. 98*2
5. 50
6. (98-2)/2 = 48
7. 14^2 + 48^2 = 50^2

15. Generalization of Pythagorean triplet finding method for equation : $A_1^2+A_2^2+....+An^2 = B^2$.

We have, $14^2 + 48^2 = 50^2$ from previous example.

- ⇨ $14^2 + 48^2 + 120^2 = 50^2 + 120^2 = 130^2$

So, we have, $14^2 + 48^2 + 120^2 = 130^2$
With this process continuing I am sure one can find the solution of
$a^2 + b^2 + c^2 = d^2$
And hence...
..
..
..
$A1^2 + A2^2 + ... + An^2 = B^2$.

16. Only square numbers have odd number of factors.

Proof : Whenever a number divides a number a quotient appears.

- ⇨ There is always an even number of factor for any number.

What if the quotient is same as the divider.
This case appears only in case of square numbers.
Let a be any number.
Now if we divide a^2 by a then the quotient is also a.

- ⇨ In case of divider is a the number of factor is 1.

Rest all appears as two numbers : one is divider and another is quotient.

- ⇨ There is even number of factors + 1 (for a)
- ⇨ There is always odd number of factors of any square number.

Proved.

17. The odd number table to handle problems on prime number.

Let p be any odd number.
We will form a table of column 3 and infinite rows of odd numbers as below :

p	p+2	p+4
p+6	p+8	p+10
p+12	p+14	p+16
p+18	p+20	p+22
p+24	p+26	p+28
p+30	p+32	p+34
p+36	p+38	p+40
p+42	p+44	p+46
p+48	p+50	p+52
p+54	p+56	p+58
p+60	p+62	p+64
p+66	p+68	p+70
p+72	p+74	p+76
p+78	p+80	p+82
p+84	p+86	p+88
p+90	p+92	p+94
p+96	p+98	p+100

.....

.....

Properties of this table :

1) All the numbers of any one column is divisible by 3.
2) The numbers which are divisible by q(q is prime) occurs in any column after q numbers in that column. For example if q is 5. Then the numbers which are divisible by 5 occurs after 5 rows in the same column. For example, if p+50 is divisible by 5 then p+80 will be the next number, in the same column, which is divisible by 5.

Conjecture

If a problem can be understood with knowledge set A then it can be solved with knowledge set A, where A is any subset of a given subject.

REFERENCES

[1] Halberstam, H.; Richert, H.-E. Sieve methods. London Mathematical Society Monographs, No. 4. Academic Press [A subsidiary of Harcourt Brace Jovanovich, Publishers], London-New York, 1974.

[2] Havil, Julian, Gamma, Exploring Euler's constant. With a foreword by Freeman Dyson. Reprint of the 2003 edition [MR1968276]. Princeton Science Library. Princeton University Press, Princeton, NJ, 2009.

[3] H. Iwaniec, Almost-primes represented by quadratic polynomials. Invent.Math. 47 (1978), no. 2, 171188.

[4] Dominic Klyve, Explicit Bounds on Twin Primes and Bruns Constant, A Thesis Submitted to the Faculty in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Mathematics

[5] Edmund Landau, Elementary number theory. Translated by J. E. Goodman.Chelsea Publishing Co., New York, N.Y., 1958.

[6] E. Landau, Collected works, Thales Verlag, Essen, 1987.

[7] Leveque, Fundamentals of Number Theory

[8] Thomas R. Nicely, Enumeration to 1014 of the Twin Primes and Brun's Constant, Virginia Journal of Science, Volume 46, Number 3, Fall 1996

[9] Niven, Ivan; Zuckerman, Herbert S.; Montgomery, Hugh L. An introduction to the theory of numbers. Fifth edition. John Wiley & Sons, Inc., New York, 1991.

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